Multi-messenger astrophysics

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1) Fermi shock acceleration for pedestrians (→ Lecture 1)

Consider that particles gain energy per cycle $E \rightarrow \eta\, E$ with $\eta > 1$ and escape per cycle with a probability $P_{\text{esc}} < 1$. Derive the differential (in energy) number density as a function of energy.

Hint: Consider energy and number of particles after n cycles.

2) Particle acceleration in steady state (effective treatment) (→ Lecture 1)

Consider particle acceleration with the acceleration rate $t_{\text{acc}}^{-1} = \eta c / R_L$, where R_L is the Larmor radius $\propto E$. The escape rate is given by $t_{\text{esc}}^{-1} \equiv E^{-\alpha}/T_0$. The injection is a δ-function describing injection at the low E_0: $Q = Q_0 \cdot \delta(E - E_0)$.

How does $N(E)$ in the steady state dependent on energy? Consider different values of α. How does the escape spectrum from the acceleration region look like for $\alpha = 1$?

Hint: Use the closed analytical solution for the steady state. The acceleration rate can be treated just as a cooling rate with inverted sign.

3) Photo-hadronic interaction rate (→ Lecture 2)

The interaction rate of initial protons p with photons γ is given by

$$\Gamma(E_p) = \int d\varepsilon \int \frac{d\cos\theta_{p\gamma}}{2} \, (1 - \cos\theta_{p\gamma}) \, n_\gamma(\varepsilon, \cos\theta_{p\gamma}) \, \sigma(\epsilon_r) \, . \tag{1}$$

Here $n_\gamma(\varepsilon, \cos\theta_{p\gamma})$ is the photon density as a function of photon energy ε and the angle between the photon and proton momenta $\theta_{p\gamma}$ ($\theta_{p\gamma} = \pi$ for heads-on collisions), $\sigma(\epsilon_r)$ is the photo-hadronic interaction cross section, and

$$\epsilon_r = \frac{E_p \varepsilon}{m_p} (1 - \cos\theta_{p\gamma}) \tag{2}$$

is the photon energy in the proton rest frame (PRF) – the cross sections are typically tabulated in. The interaction itself, and therefore E_p and ε, are to be described in the shock rest frame (SRF).

a) What is the relationship between ϵ_r and the center-of-mass energy $\sqrt{s}$, where $s = (P_1 + P_2)^2$ and P_1 and P_2 are the relativistic 4-momenta of proton and photon, respectively?

Bonus question: Once you are at this point, you can also easily derive the relationship Eq. (2).

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b) Assume isotropically distributed target photons, *i.e.*, $n_\gamma(\varepsilon, \cos\theta_{p\gamma}) = n_\gamma(\varepsilon)$. Re-write Eq. (1) in the form

$$\Gamma(E_p) = \int d\varepsilon\, n_\gamma(\varepsilon)\, F(y) \tag{3}$$

with $y \equiv E_p\,\varepsilon/m_p$. What is the meaning of $F(y)$ and y?

c) Compute and plot/discuss the function $F(y)$ for the Δ-resonance box function approximation ("LR" for "lower resonance" in the following plot):