GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

HELMHOLTZ ASSOCIATION
Alliance for Astroparticle Physics

Particle Physics

Tutorial (Solutions)

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Problem 1: LHC at $\sqrt{s} = 13\,\mathrm{TeV}$

a) Two gluons (of the incoming protons) collide head-on. Each carries 10% of the momentum of its proton. Compute the cm energy of the two gluons.

$$p_1 = \begin{pmatrix} xE_p \\ xE_p\vec{e}_z \end{pmatrix} \qquad p_2 = \begin{pmatrix} xE_p \\ -xE_p\vec{e}_z \end{pmatrix} \quad \text{with } x = 0.1,\ E_p = \frac{\sqrt{s}}{2}$$

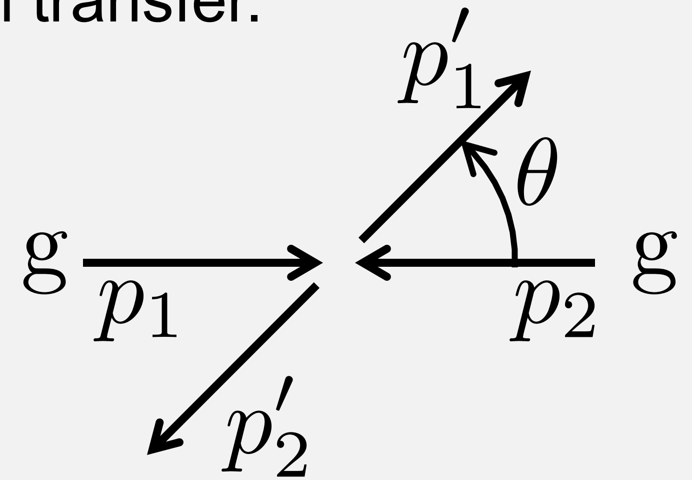
$$(p_1 + p_2)^2 = \begin{pmatrix} 2xE_p \\ \vec{0} \end{pmatrix}^2 = 4x^2E_p^2 = x^2 s$$

$$\sqrt{\hat{s}} = \sqrt{(p_1 + p_2)^2} = x\sqrt{s} = 0.1 \times 13\,\mathrm{TeV} = 1.3\,\mathrm{TeV}$$

Problem 1: LHC at $\sqrt{s} = 13\,\mathrm{TeV}$

b) The gluons are scattered by an angle $\theta = 45^\circ$. Compute q^2, the square of the four-momentum transfer.

$$p_1 = \begin{pmatrix} xE_\mathrm{p} \\ 0 \\ 0 \\ xE_\mathrm{p} \end{pmatrix} \qquad p_1' = \begin{pmatrix} xE_\mathrm{p} \\ 0 \\ xE_\mathrm{p}\sin\theta \\ xE_\mathrm{p}\cos\theta \end{pmatrix}$$



$$q = p_1' - p_1 = \begin{pmatrix} 0 \\ 0 \\ xE_\mathrm{p}\sin\theta \\ xE_\mathrm{p}(\cos\theta - 1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.46 \\ -0.19 \end{pmatrix} \mathrm{TeV}$$

$$q^2 = \left(-0.46^2 - (-0.19)^2\right)\mathrm{TeV}^2 = -0.25\,\mathrm{TeV}^2$$

Problem 1: LHC at $\sqrt{s} = 13\,\mathrm{TeV}$

c) Compute the spatial resolution $\hbar/\sqrt{|q^2|}$ reached in this scattering process. Note: $1 = \hbar c = 0.2\,\mathrm{GeV\,fm}$

$$\delta r = \frac{\hbar}{\sqrt{|q^2|}} = \frac{\hbar c}{\sqrt{|q^2|}} = \frac{0.2\,\mathrm{GeV\,fm}}{\sqrt{0.25}\,\mathrm{TeV}} = 4 \times 10^{-4}\,\mathrm{fm}$$

d) A proton of one of the LHC beams hits a proton inside a gas molecule left in the (highly evacuated) beam pipe. Compute the cm energy for this collision.

$$\sqrt{s} = \sqrt{2 m_\mathrm{p} E_\mathrm{p}} = \sqrt{2 \times 0.938\,\mathrm{GeV} \times 6.5\,\mathrm{TeV}} = 110\,\mathrm{GeV}$$

Problem 2: Future circular collider?

The LHC has a circumference of 27 km. With the current superconducting magnets it can reach a maximum cm energy (proton-proton) of 14 TeV with 3×10^{14} protons per beam.

a) Consider a future LHC-like machine with a cm energy of 100 TeV. Assume that by aggressive R&D one can double the field of the magnets. Find the circumference of the ring.

$$R = \frac{\gamma m v}{eB} \overset{v\approx c=1}{\approx} \frac{\gamma m}{eB}$$

$$\frac{C_{\rm FCC}}{C_{\rm LHC}} = \frac{E_{\rm FCC} B_{\rm LHC}}{E_{\rm LHC} B_{\rm FCC}} = \frac{100\times 1}{14\times 2} = 3.57$$

$$C_{\rm FCC} = 3.57\times 27\,{\rm km} = 96\,{\rm km}$$

Problem 2: Future circular collider?

The LHC has a circumference of 27 km. With the current superconducting magnets it can reach a maximum cm energy (proton-proton) of 14 TeV with 3×10^{14} protons per beam.

b) The synchrotron radiation power of one LHC beam (at design) is 6 kW. What would be the corresponding power for the new machine assuming the same number of particles per length?

$$P \propto N\gamma^4 R^{-2} \propto N E^4 C^{-2}$$

$$N \propto C \quad \Rightarrow \quad P \propto E^4 C^{-1}$$

$$\frac{P_{\mathrm{FCC}}}{P_{\mathrm{LHC}}} = \left(\frac{E_{\mathrm{FCC}}}{E_{\mathrm{LHC}}}\right)^4 \times \frac{C_{\mathrm{LHC}}}{C_{\mathrm{FCC}}} = \left(\frac{100}{14}\right)^4 \times \frac{1}{3.57} = 729$$

$$P_{\mathrm{FCC}} = 729 \times 6\,\mathrm{kW} = 4.4\,\mathrm{MW}$$

Problem 2: Future circular collider?

The LHC has a circumference of 27 km. With the current superconducting magnets it can reach a maximum cm energy (proton-proton) of 14 TeV with 3×10^{14} protons per beam.

c) Consider now an electron-positron collider in the same tunnel, able to produce top-quark pairs (cm energy 350 GeV). Compute the synchrotron radiation power if 100 bunches, each with 10^{11} particles are stored per beam.

$$P = N \times \frac{q^2}{6\pi}\gamma^4 R^{-2} \propto \frac{N}{C^2}\left(\frac{E}{m}\right)^4$$

$$N_\mathrm{p} = 3 \times 10^{14} \times \frac{C_\mathrm{FCC}}{C_\mathrm{LHC}} = 1.1 \times 10^{15}$$

$$\frac{P_\mathrm{e}}{P_\mathrm{p}} = \frac{N_\mathrm{e}}{N_\mathrm{p}}\frac{C_\mathrm{p}^2}{C_\mathrm{e}^2}\left(\frac{E_\mathrm{e} m_\mathrm{p}}{E_\mathrm{p} m_\mathrm{e}}\right)^4 = \frac{10^{13}}{1.1 \times 10^{15}} \times 1 \times \left(\frac{0.350 \times 938}{100 \times 0.511}\right)^4 = 15.9$$

$$P_\mathrm{e} = 15.9 \times P_\mathrm{p} = 70\,\mathrm{MW}$$

Problem 3: What is the correct pronunciation of:

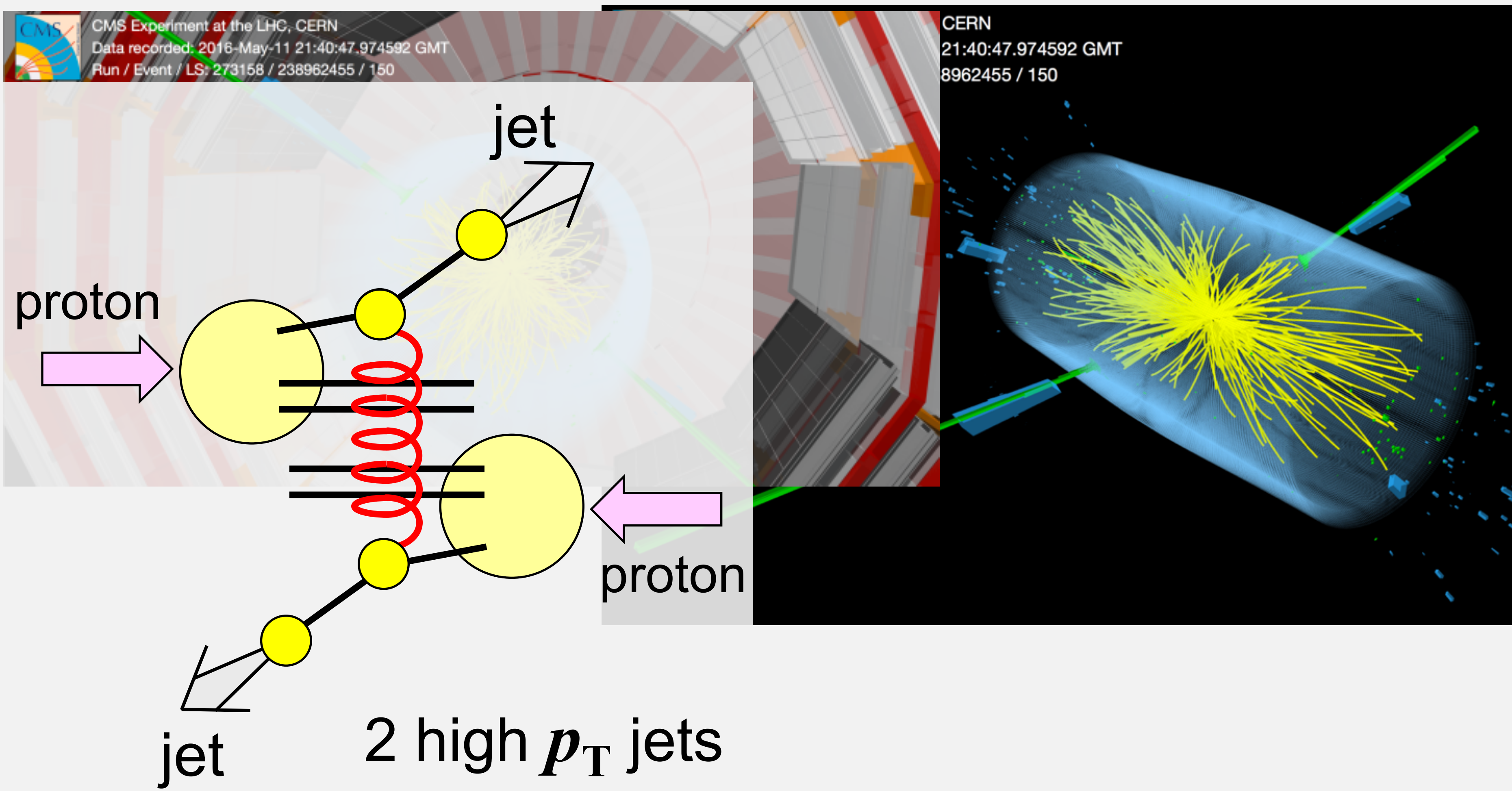
Richard P. Feynman

Richard P. “FineMan“

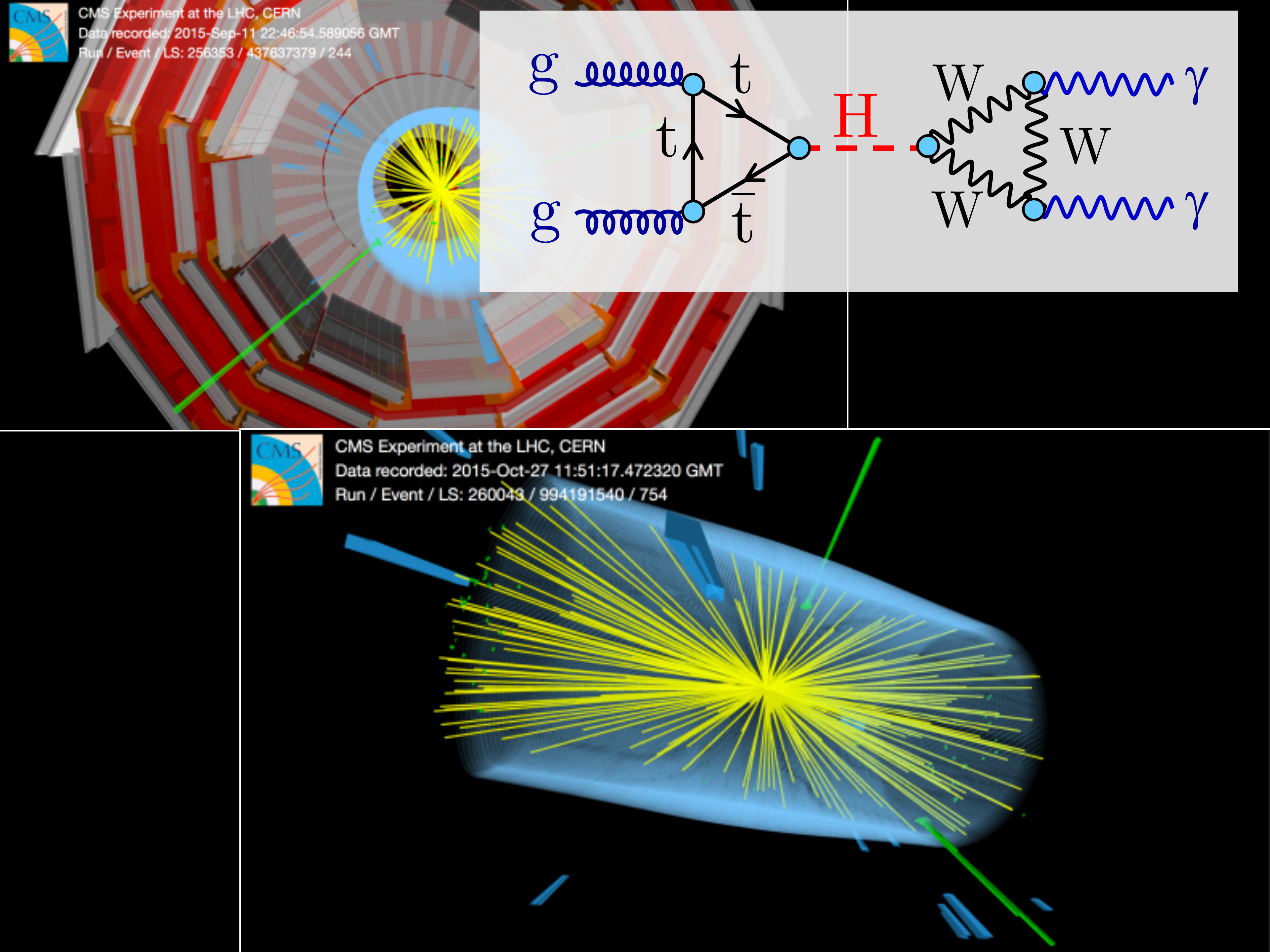
faɪnmən

Problem 4: Event quizz

Look at the following event displays and identify final state objects. Can you imagine which process could have taken place? If possible, draw a Feynman diagram.



2 high p_T jets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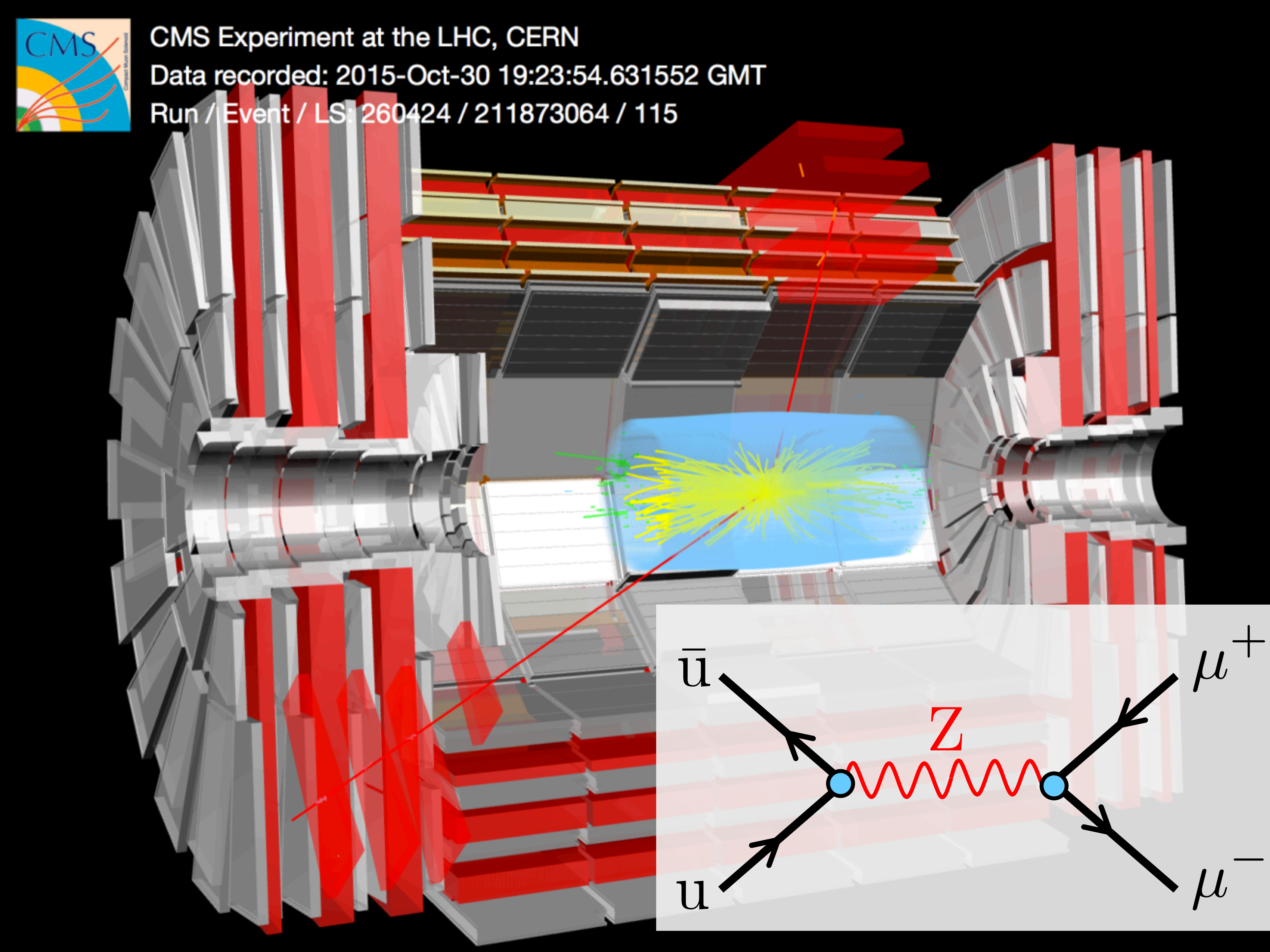
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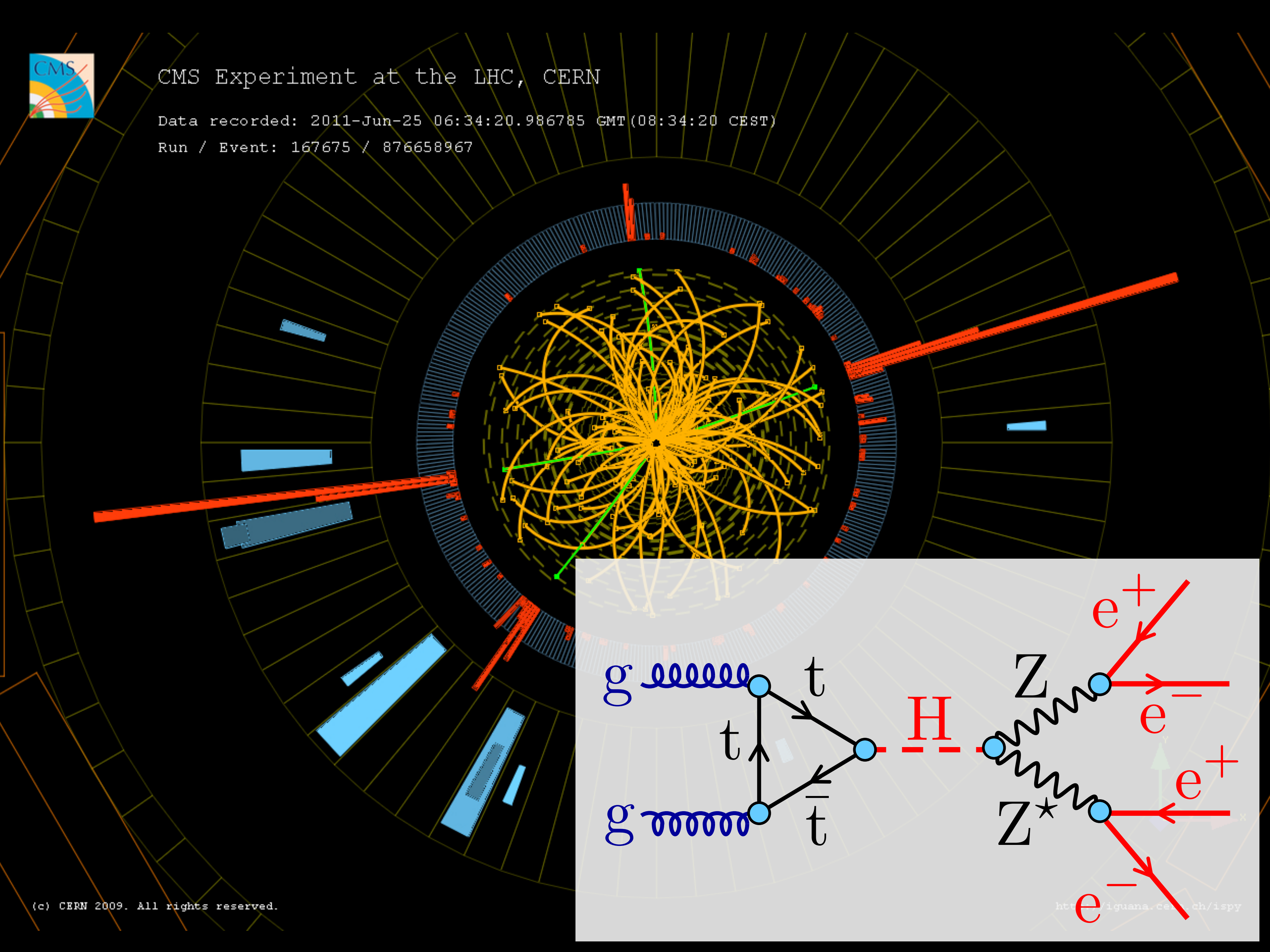
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(c) CERN 2009. All rights reserved.
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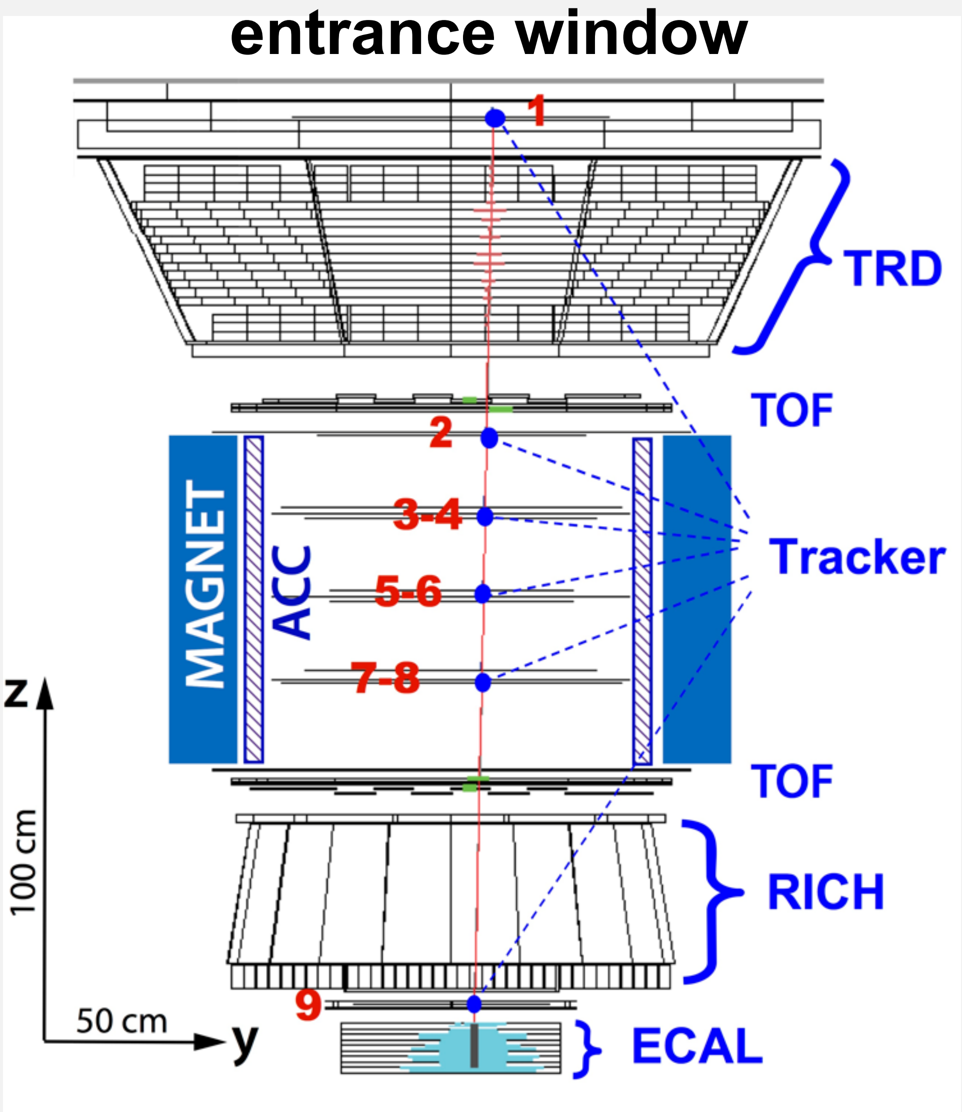
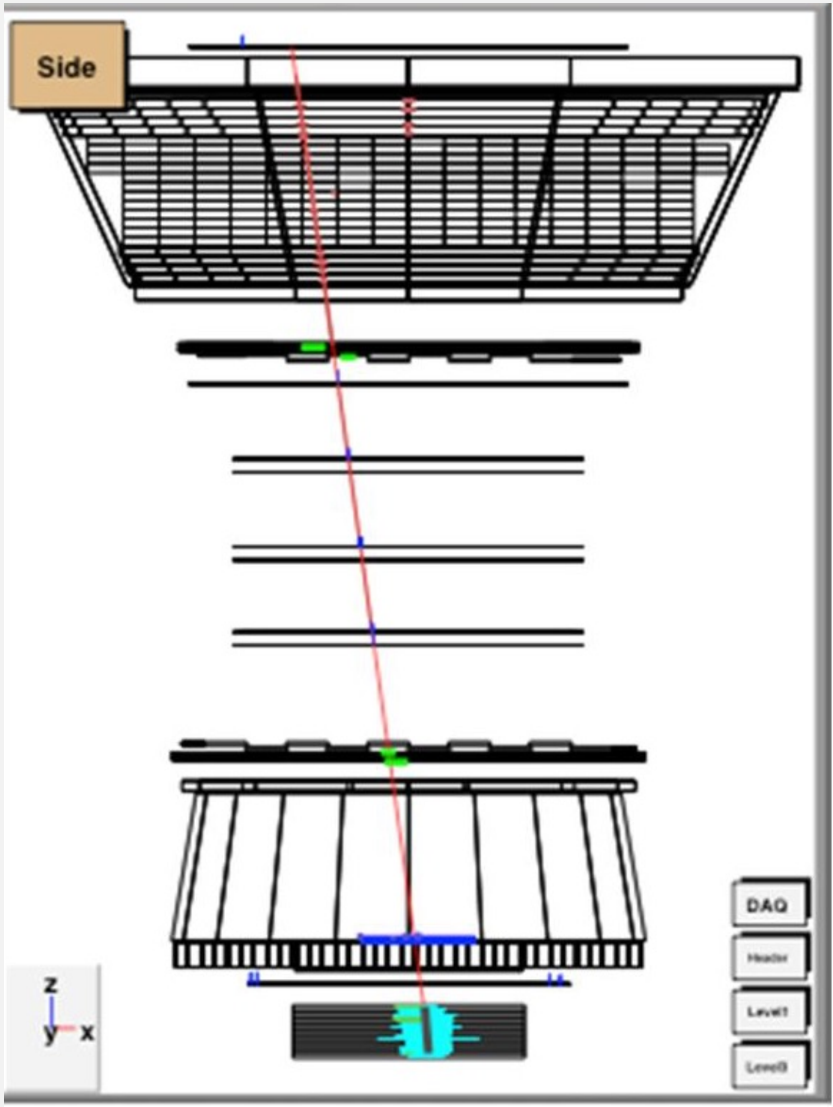


incoming electron (or positron), absorbed in ECAL



Problem 6: Fill all charge numbers (if applicable) into the following table.

particle	Q	I	I_3	Y	colored? y/n
e_L^-	−1	½	−½	−1	n (singlet)
$\nu_{\mu L}$	0	½	+½	−1	n (singlet)
τ_R^-	−1	0	0	−2	n (singlet)
t_L	+⅔	½	+½	+⅓	y (triplet)
b_R	−⅓	0	0	−⅔	y (triplet)
$\nu_{\tau R}$	0	0	0	0	n (singlet)
γ	0	0&1	0	0	n (singlet)
Z	0	0&1	0	0	n (singlet)
W^-	−1	1	−1	0	n (singlet)
g	0	0	0	0	y (octet)

Problem 7: WW scattering

In the lecture we discussed all diagrams (of leading order) contributing to the process:

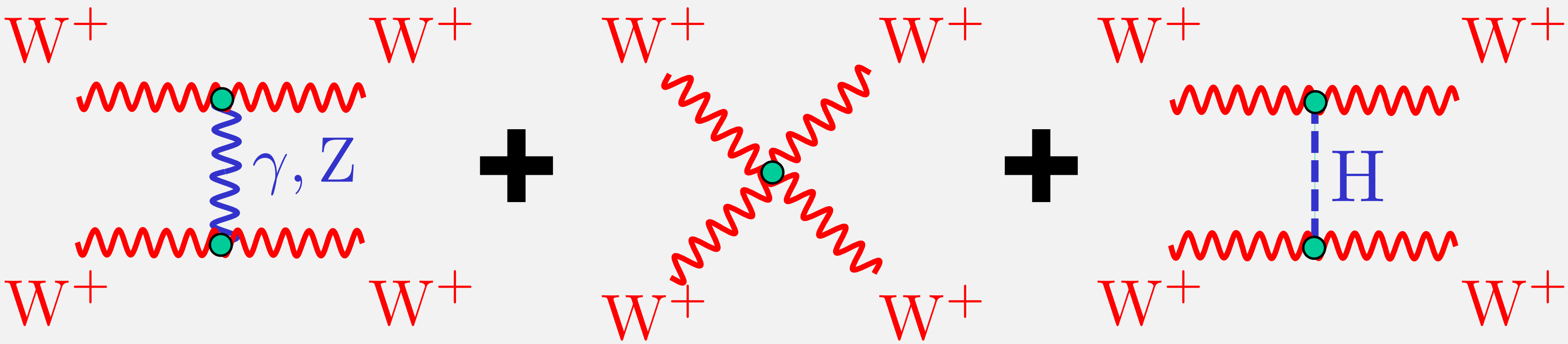
$$W^+W^- \rightarrow W^+W^-$$

Draw all leading order diagrams contributing to the related process:

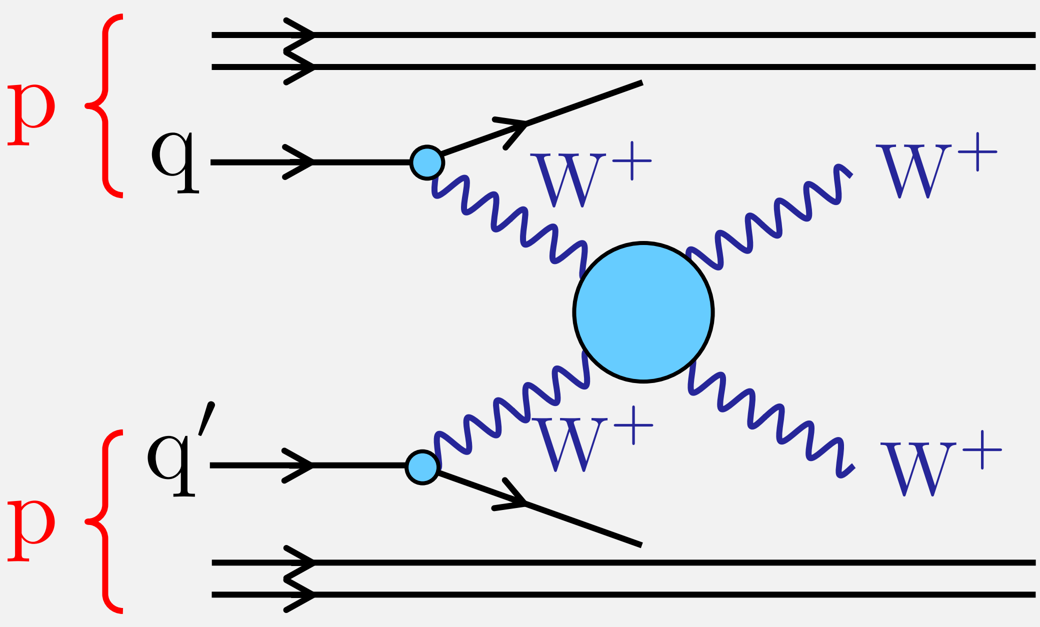
$$W^+W^+ \rightarrow W^+W^+$$

Draw a diagram which shows, how this scattering could happen at the LHC in proton-proton collisions.

$$W^+W^+ \to W^+W^+$$



At the LHC:



- 2 (leptonic) W^+ decays
- 2 high p_T forward jets