Fast Single-Top Cross Section Predictions for Hadron Colliders with the HATHOR Program

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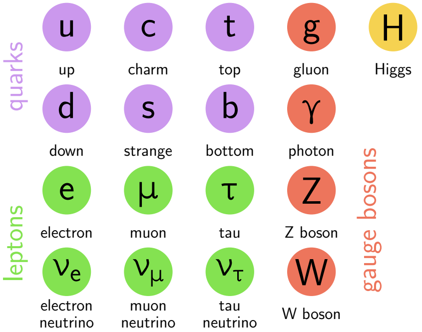
Outline

The Top Quark in the Standard Model

- Standard model: fundamental interactions of elementary particles
- $m_t = 173.5\,\text{GeV}$, charge $+\frac{2}{3}e$
- Short lifetime: $5 \times 10^{-25}\,\text{s}$
- Typical decay: $t \to Wb$
- Direct measurement of $|V_{tb}|^2$
- Suitable laboratory for new physics: e.g. anomalous couplings



Requirements:

- Precise knowledge of the cross section and its dependences: scales, parton densities, couplings, masses, ...
- Quick calculation desireable!

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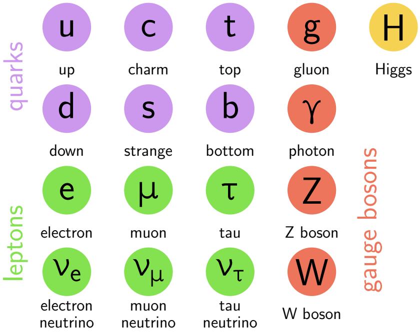

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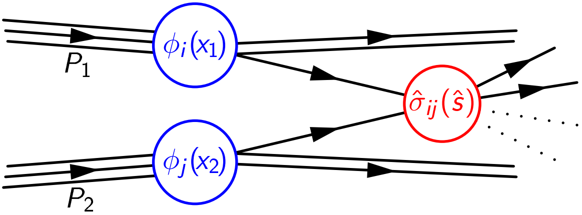
Calculating Hadronic Cross Sections

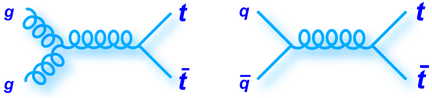
Factorisation into PDFs and partonic cross section:



$$\sigma_{\text{had}} = \sum_{ij} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, \phi_{i,\mathrm{h}_1}(x_1, \mu_\mathrm{F})\, \phi_{j,\mathrm{h}_2}(x_2, \mu_\mathrm{F})\, \hat{\sigma}_{ij}(\hat{s}, \mu_\mathrm{F})$$

- $\phi(x, \mu_\mathrm{F})$ parton densities
- $\hat{\sigma}_{ij}(\hat{s}, \mu_\mathrm{F})$ partonic cross section
- $\hat{s} = x_1 x_2 s$ part. centre-of-mass energy
- $x_{1,2}$ fractional parton momentum
- μ_F factorisation scale

- Top anti-top pairs produced by strong interaction

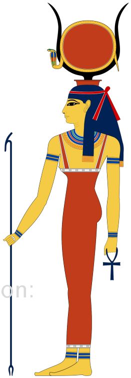


- Implemented in the HATHOR program
  - Fast calculation of hadronic cross sections for $t\bar{t}$ production:
    [M. Aliev et al, Comput.Phys.Commun.182: 1034-1046, 2011]
  - Provides reference values in NNLO-QCD:

$$\hat{\sigma} = \underbrace{\alpha_s^2\,\hat{\sigma}^{\mathrm{LO}}}_{\substack{\text{leading}\\ \text{order}}} + \underbrace{\alpha_s^3\,\hat{\sigma}^{\mathrm{NLO}}}_{\substack{\text{next-to-leading}\\ \text{order}}} + \underbrace{\alpha_s^4\,\hat{\sigma}^{\mathrm{NNLO}}}_{\substack{\text{next-to-next-to-leading}\\ \text{order}}} + \mathcal{O}(\alpha_s^5)$$

  - Calculation of theoretical uncertainties (PDFs, scales, ...)

Top Pair Production



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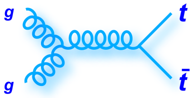

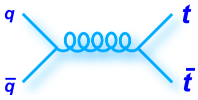

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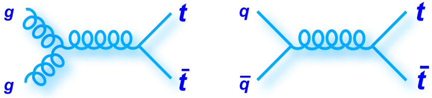

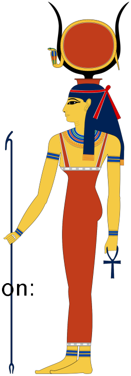

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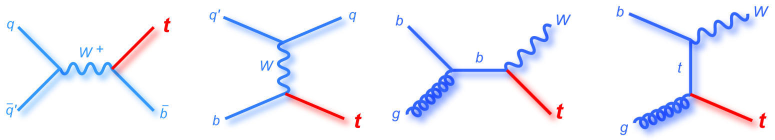
Electroweak Top-Quark Production

Production of single top-quarks:



s-channel | *t*-channel | Wt production

- NLO calculation exists in MCFM, not in HATHOR (Monte Carlo for FeMtobarn processes)
- Long running: $\mathcal{O}$(1h) per hadronic cross section
- Studying theoretical uncertainties: repeated execution necessary, requires $\mathcal{O}$(days)

$\Rightarrow$ Desireable:
Faster calculation of total hadronic cross sections

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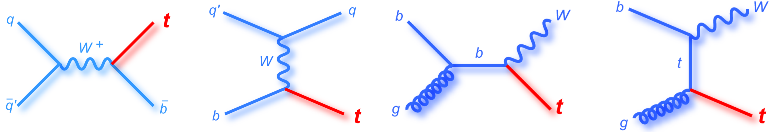

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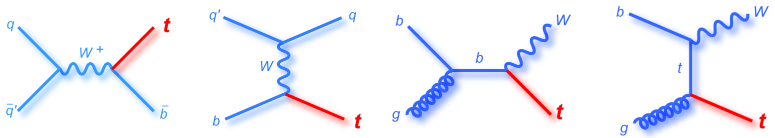

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Extraction of Partonic Cross Sections

$$\sigma_{\text{had}}(s) = \sum_{ij} \int \mathrm{d}x_1\, \mathrm{d}x_2\, \phi_{i,\text{h}_1}(x_1, \mu_F)\, \phi_{j,\text{h}_2}(x_2, \mu_F)\, \hat{\sigma}_{ij}(\underbrace{x_1 x_2 s}_{\hat{s}}, \mu_F)$$

For $t\bar{t}$ production:

- $\hat{\sigma}$ has closed-form expression (function of $\hat{s}$)

For single-top quark production:

- Code for $\hat{\sigma}$ does not exist in closed-form
- $\hat{\sigma}(\hat{s})$ implemented in MCFM as follows:

$$\hat{\sigma} = \underbrace{\int_n \mathrm{d}\sigma_{\text{LO}}}_{\text{easy, finite}} + \underbrace{\int_n \mathrm{d}\sigma_{\text{virt.}} + \int_{n+1} \mathrm{d}\sigma_{\text{real}} + \int_{n+1} \mathrm{d}\sigma_{\text{fact.}}}_{\text{infrared/collinear divergences in each part}}$$

- Integration of real and virtual corrections on different phase spaces $\Rightarrow$ Cancellation of divergences is not trivial

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Extraction of Partonic Cross Sections (I)

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- Target: Extract $\hat{\sigma} = \int \mathrm{d}\hat{\sigma}(\hat{s})$
- Idea: Fix the partonic centre-of-mass energy $\hat{s}$ in integration
- Minimal-invasive approach: replace PDFs with delta functions

$$\phi(x) \xrightarrow{\textit{insert}} \delta(x - x_0) \approx \frac{1}{\sqrt{2\pi w}} \exp\left(-\frac{(x - x_0)^2}{2w^2}\right)$$

- MCFM calculates effectively $\hat{\sigma}$ at arbitrary position $x_0^2 s$

✓ Simple and quick implementation

✗ Inefficient in numerical integration (esp. at $w \to 0$)

✗ Systematic error due to finite width w of PDFs

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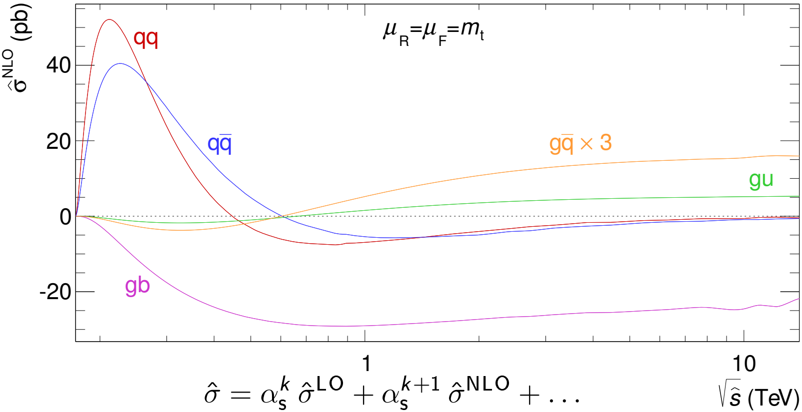
Extraction of Partonic Cross Sections (I)

Result: NLO contributions in single-top t-channel



$$\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{LO} + \alpha_s^{k+1} \, \hat{\sigma}^{NLO} + \dots$$

Timespan: several weeks for all processes!

Extraction of Partonic Cross Sections (II)

Alternative approach:

- Identification of partonic centre-of-mass energy in MCFM
- Insert exact delta functions:

$$\hat{\sigma}(\hat{s}) = \int \delta\left(x_1 - x_0\right) \delta\left(x_2 - x_0\right) \mathrm{d}\hat{\sigma}(x_1 x_2 s) \, \mathrm{d}x_1 \, \mathrm{d}x_2$$

- Adjustments to $x_{1,2}$ directly in MCFM code

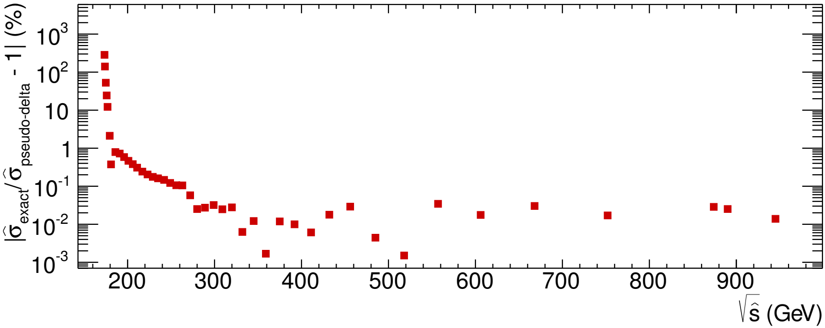
✓ Very efficient extraction von $\hat{\sigma}$

✓ No inherent, systematic error

✗ Invasive modifications to the code necessary

Comparison of Both Methods

Relative difference of both methods in t-channel ($ub \to tq$)

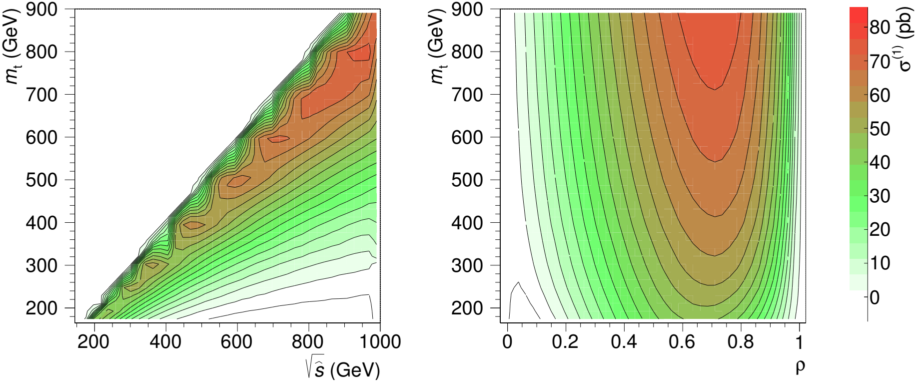


- Successful consistency check, very good agreement
- First method's issues at threshold already known

$\Rightarrow$ Using this (2nd) method:
Extraction as function of $\hat{s}$ and m_t possible

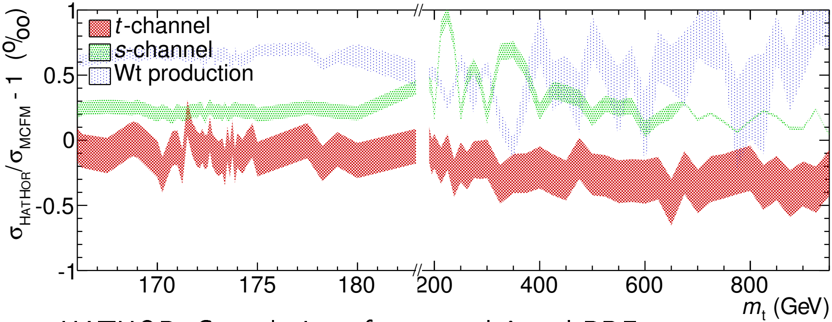
Partonic Cross Section

- Extraction of $\hat{\sigma}$ for many sampling points $\hat{s}$ und m_t
- Implementation of $\hat{\sigma}$ in HATHOR
- Interpolation of $\hat{\sigma}(\hat{s}, m_t)$ on 2D grid necessary ($\rho = m_t^2/\hat{s}$ helps)



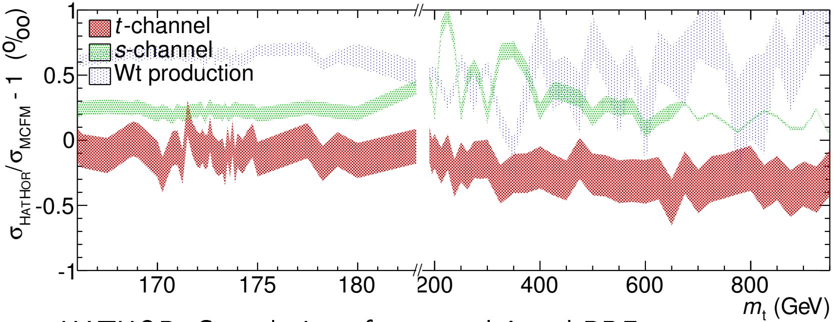
- Integration of $\hat{\sigma}$ and PDFs in HATHOR $\Rightarrow \sigma_{had}$!

Consistency Check



- HATHOR: Convolution of extracted $\hat{\sigma}$ and PDFs
- Comparison of σ_{had} with complete MCFM calculation
- Precision of $\mathcal{O}(0{,}1\%)$ over large mass range

Time required:
- HATHOR: $\sim$ 1 hour
- MCFM: $\sim$ 1 week



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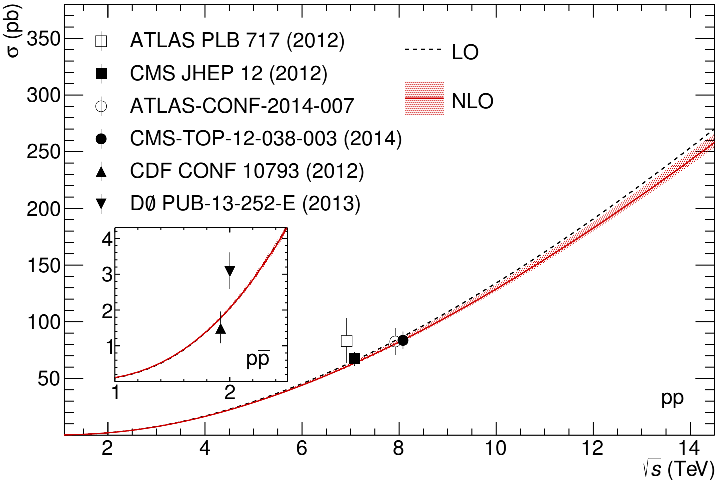
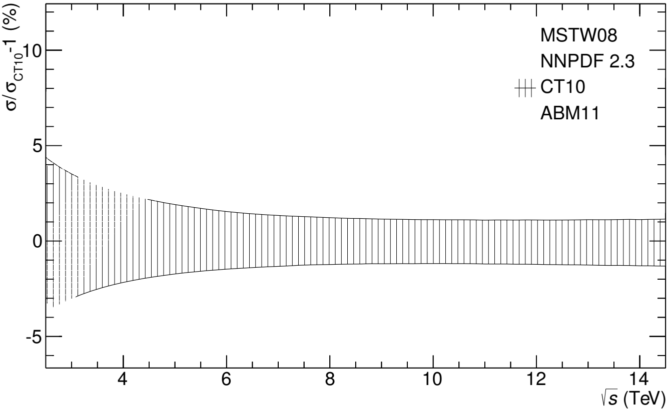
Hadronic Cross Section: t-channel



- Different orders of perturbation series in t-channel

Impact of PDFs in NLO



- PDF uncertainties in NLO (t-channel)
- Time consuming: $\sim$ 100 error PDFs
- Errors from varying PDF fit parameters and α_s
- Error band does not always cover differences

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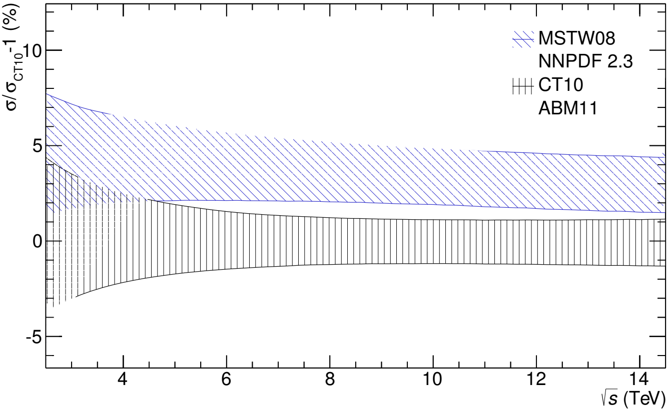

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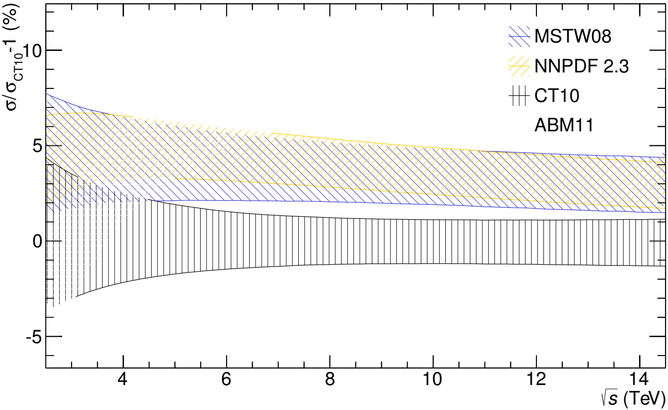

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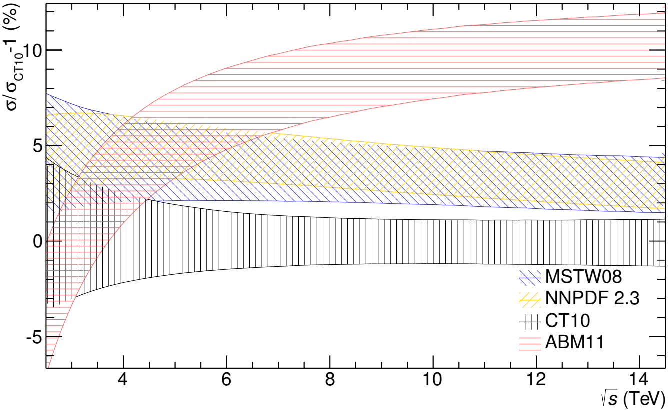

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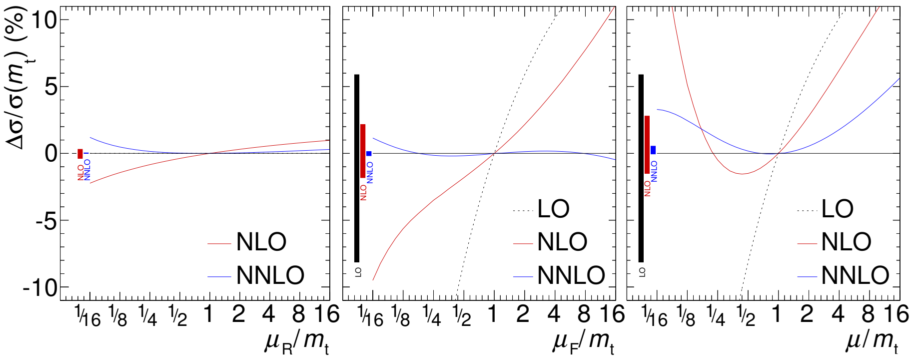
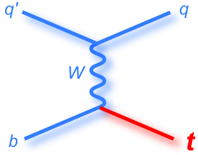
Scale Dependence of the Cross Section



- Scale dependence in the *t*-channel (exact calculation by S. Mölbitz)
- Cross section normalized to natural scale ($\mu = m_t$)
- Dependences in higher orders considerably reduced



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Summary

Extraction of partonic cross sections:

- ✓ Implemented two methods
- ✓ Consistency checks using $t\bar{t}$ production
- ✓ Applied to production of single top-quarks in NLO

Implementation in HATHOR:

- ✓ Reached precision of $\leq \mathcal{O}(0.1\%)$
- ✓ Much better performance than e.g. MCFM
- ✓ Exact scale dependence in NLO und NNLO

Demonstrated abilities of the implementation:

- ✓ Study of systematic uncertainties (PDFs, scales, CKM, ...)
- ✓ Possibility to produce plots in $\mathcal{O}(1\,\mathrm{h})$
- ✓ Application to top-quark mass measurement

Release:

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  - ▶ Preprint online (arXiv:1406.4403), publication in progress

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Thank you for your attention!

Backup

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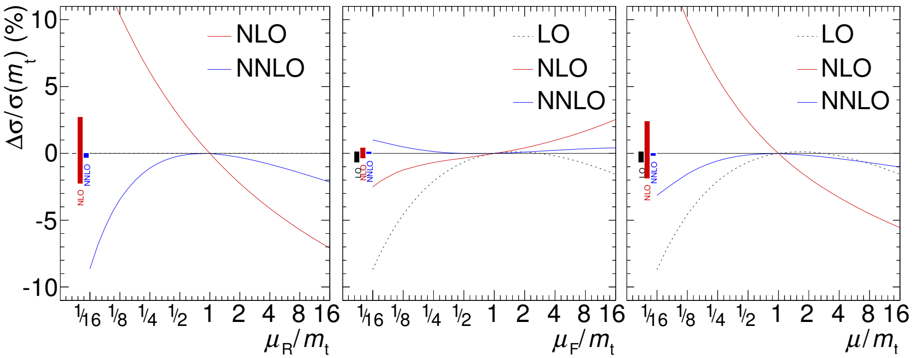
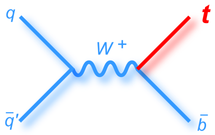
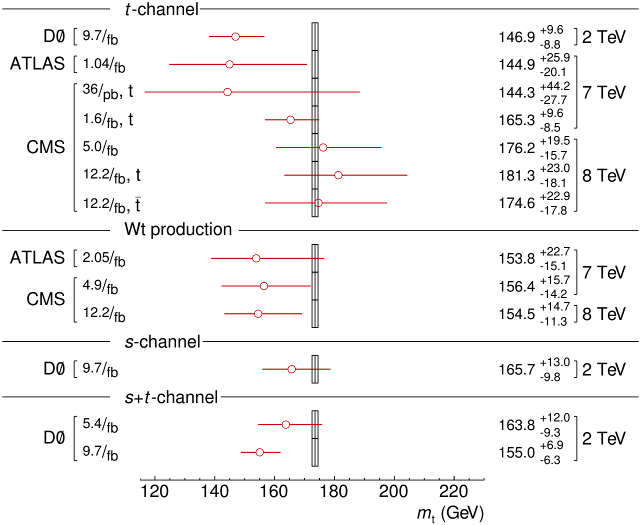


- Scale dependence in the *s*-channel (exact calculation by S. Mölbitz)
- Cross section normalized to natural scale ($\mu = m_t$)
- Underlines necessity of higher-order calculations



Determination of the Top-Quark Mass



- Determining the top-quark mass from cross sections in NLO
- Correlations neglected
- Highest sensitivity in *s*-channel
- Lowest sensitivity in *t*-channel

Implementation of Scale Dependences

- Expand partonic cross section:

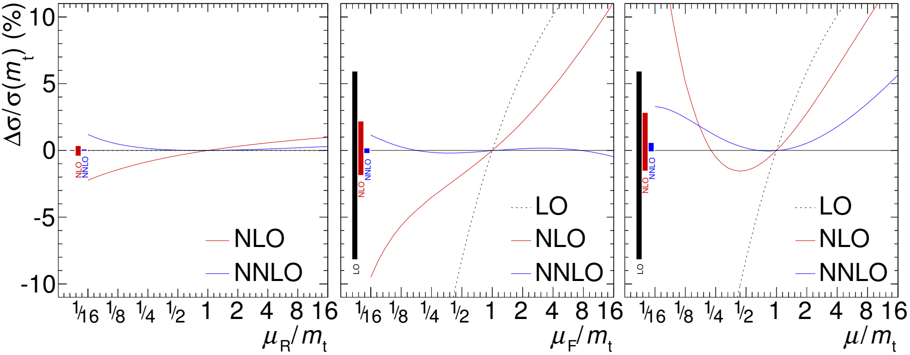
$$\hat{\sigma} = \alpha_s^k \hat{\sigma}^{LO} + \alpha_s^{k+1} \hat{\sigma}^{NLO} + \dots$$

- Extraction of $\hat{\sigma}^{NLO}$ at a scale $\mu_F = \mu_R = m_t$
- Extension to arbitrary scales:

$$\hat{\sigma}^{NLO} = \hat{\boldsymbol{\sigma}}^{(\mathbf{10})} + \log\left(\frac{\mu_F^2}{m_t^2}\right)\hat{\sigma}^{(11)} + k\beta_0 \log\left(\frac{\mu_F^2}{\mu_R^2}\right)\hat{\sigma}^{LO}$$

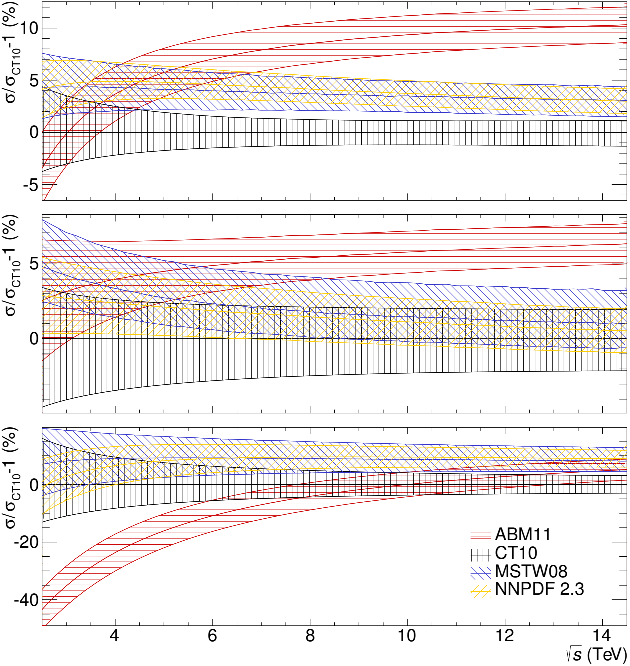
- Implement $\hat{\boldsymbol{\sigma}}^{(\mathbf{10})}$ using extracted cross sections
- Calculation of $\hat{\sigma}^{(11)}$ in master thesis of S. Mölbitz
- Allows variation of α_s and scales

Scale Dependence of the Cross Section

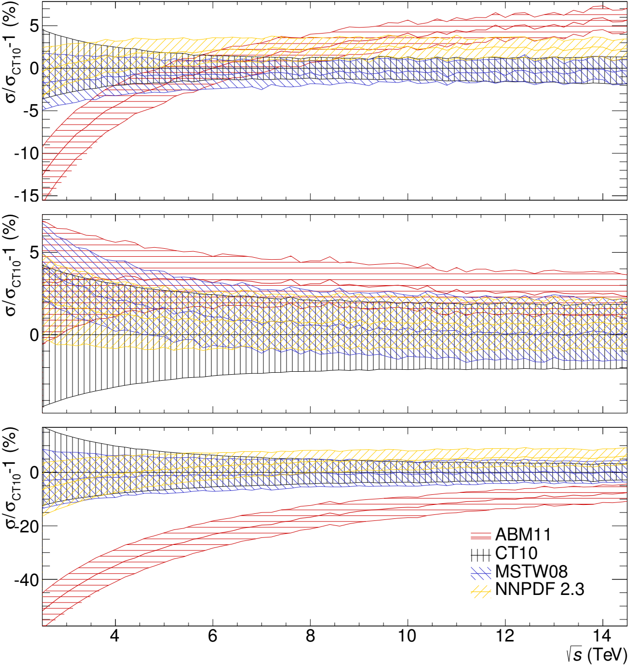


- Scale dependence in *t*-channel
  (exact calculation by S. Mölbitz)
- Very small μ_R dependence
- Dependences in NNLO considerably reduced

Impact of PDFs



- PDF uncertainties in all three channels in NLO
- Time consuming: $\sim$ 100 error PDFs
- Errors from varying PDF fit parameters and α_s
- Error band does not always cover differences



Impact of PDFs

- PDF uncertainties in all three channels in approx. NLO
- Time consuming: $\sim$ 100 error PDFs
- Errors from varying PDF fit parameters and α_s
- Error band covers differences better than in NLO

$$\sigma_{\text{had}}(S) = \sum_{ij} \iint\limits_0^1 \mathrm{d}x_1\, \mathrm{d}x_2 \Bigg\{ \phi_{i,\mathrm{h}_1}(x_1)\phi_{j,\mathrm{h}_2}(x_2)\ \hat{\sigma}_{ij}(\hat{s})$$

$$+ \int\limits_{x_1}^{1} \frac{\mathrm{d}z}{z}\ \phi_{i,\mathrm{h}_1}\left(\tfrac{x_1}{z}\right) \phi_{j,\mathrm{h}_2}(x_2)\ \hat{\sigma}_{ij}^{(z)}(\hat{s})$$

$$+ \int\limits_{x_2}^{1} \frac{\mathrm{d}z}{z}\ \phi_{i,\mathrm{h}_1}(x_1)\phi_{j,\mathrm{h}_2}\left(\tfrac{x_2}{z}\right)\ \hat{\sigma}_{ij}^{(z)}(\hat{s}) \Bigg\}$$

$$\sigma_{\text{had}}(S) = \sum_{ij} \iint_0^1 dx_1\, dx_2\, \phi_{i,\text{h}_1}(x_1)\phi_{j,\text{h}_2}(x_2) \, \Bigg\{ \hat{\sigma}_{ij}(\hat{s}) \; + $$

$$+ \int_0^1 dz \left[\hat{\sigma}_{ij}^{(z)}((z\, x_1)x_2 S) + \hat{\sigma}_{ij}^{(z)}(x_1(z\, x_2)S) \right] \Bigg\}$$

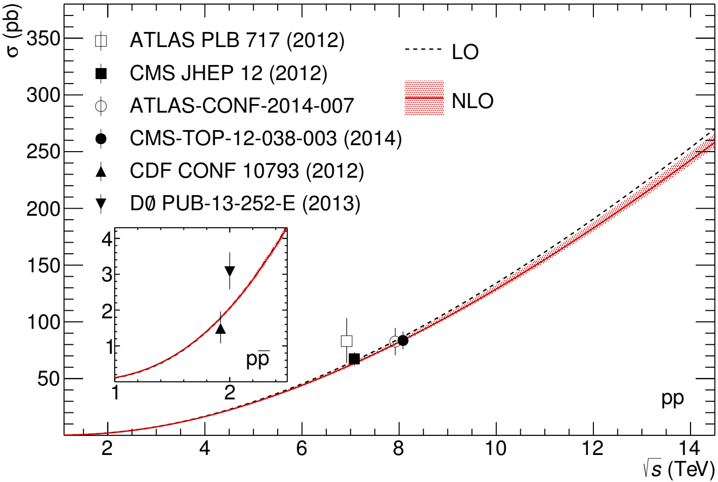
$$= \sum_{ij} \iint_0^1 dx_1\, dx_2\, \phi_{i,\text{h}_1}(x_1)\phi_{j,\text{h}_2}(x_2) \, \Bigg\{ \hat{\sigma}_{ij}(\hat{s}) \; + $$

$$+2 \int_0^1 dz\; \hat{\sigma}_{ij}^{(z)}((z\, x_1 x_2 S) \Bigg\}$$

MCFM-Integration

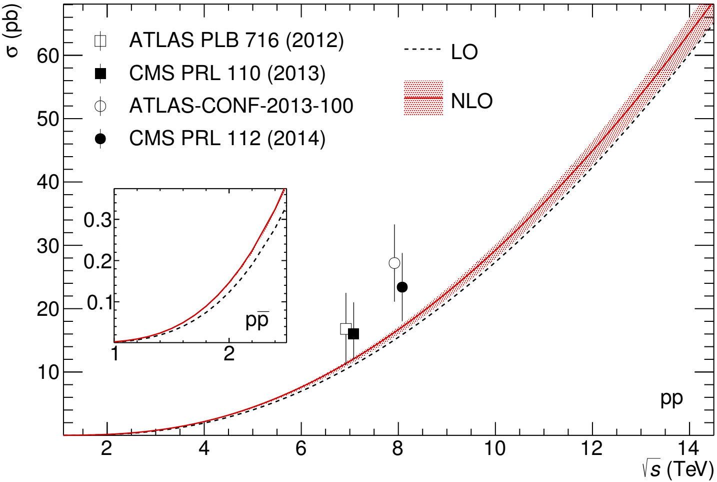
$$\hat{\sigma}_{ij}(\hat{s}) = \hat{\sigma}_{ij}(\hat{s}) + 2\int\limits_0^1 \mathrm{d}z\ \hat{\sigma}_{ij}^{(z)}(z\hat{s})$$

Hadronischer Wirkungsquerschnitt: *t*-Kanal



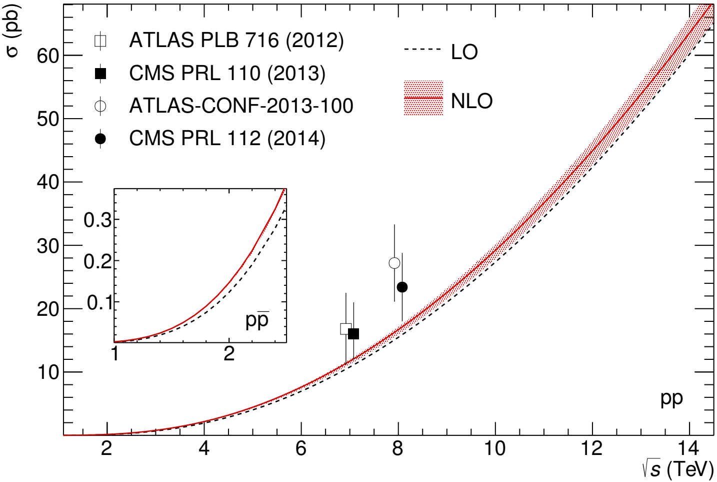
- Verschiedene Ordnungen der Störungsreihe im *t*-Kanal

Hadronischer Wirkungsquerschnitt: s-Kanal



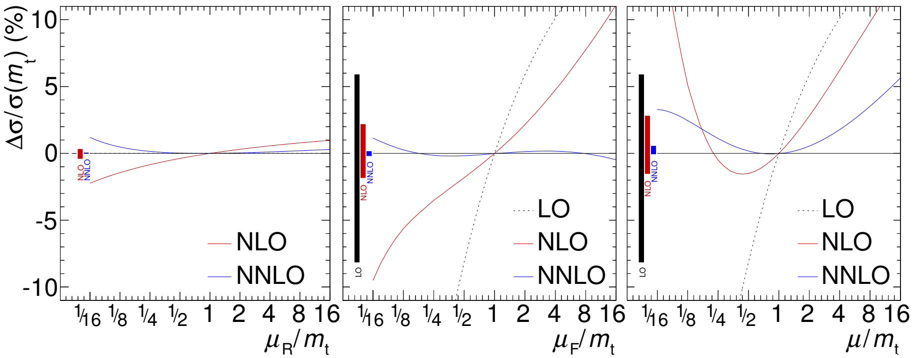
- Verschiedene Ordnungen der Störungsreihe im s-Kanal

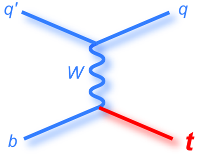
Hadronischer Wirkungsquerschnitt: Wt-Produktion



- Verschiedene Ordnungen der Störungsreihe für tW-Produktion

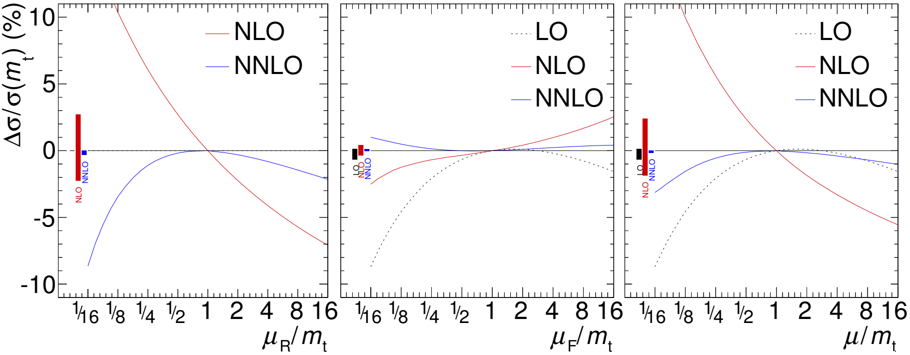
Skalenabhängigkeit des Wirkungsquerschnitts



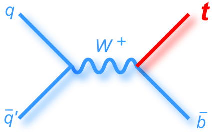


- Skalenabhängigkeiten im *t*-Kanal (exakte Rechnung von S. Mölbitz)

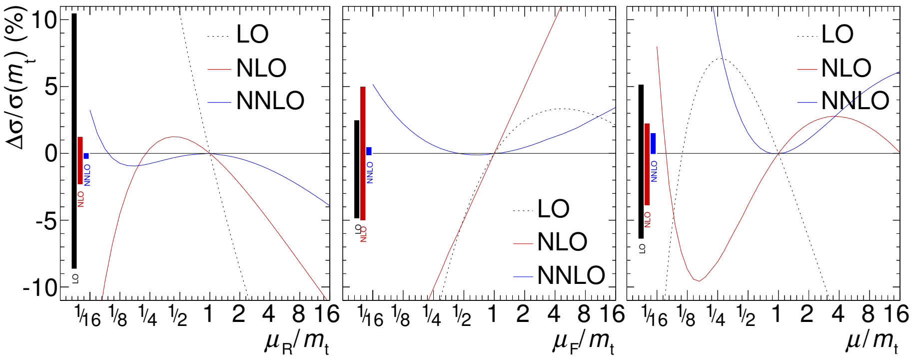
Skalenabhängigkeit des Wirkungsquerschnitts

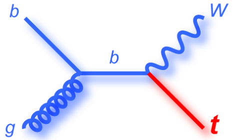


- Skalenabhängigkeiten im *s*-Kanal (exakte Rechnung von S. Mölbitz)



Skalenabhängigkeit des Wirkungsquerschnitts





- Skalenabhängigkeiten für tW-Produktion (exakte Rechnung von S. Mölbitz)

- Wirkungsquerschnitt in Potenzen von α_s:

$$\sigma = \alpha_s^k \sigma_0 + \alpha_s^{k+1} \sigma_1(\mu_R) + \alpha_s^{k+2} \sigma_2(\mu_R) + \mathcal{O}(\alpha_s^{k+3})$$

- β-Funktion der QCD:

$$\beta(\alpha_s) = \frac{\partial \alpha_s}{\partial \log \mu_R^2} = \mu_R^2 \frac{\partial \alpha_s}{\partial \mu_R^2} = -(4\pi)\beta_0 \alpha_s^2 - (4\pi)^2 \beta_1 \alpha_s^3 - \mathcal{O}(\alpha_s^4)$$

- Ableitung von σ nach μ_R:

$$\frac{\partial \sigma}{\partial \log \mu_R^2} = k\alpha_s^{k-1}\beta(\alpha_s)\sigma_0 + (k+1)\alpha_s^k \beta(\alpha_s)\sigma_1(\mu_R) + \alpha_s^{k+1} \frac{\partial \sigma_1(\mu_R)}{\partial \log \mu_R^2}$$
$$+ (k+2)\alpha_s^{k+1}\beta(\alpha_s)\sigma_2(\mu_R) + \alpha_s^{k+2} \frac{\partial \sigma_2(\mu_R)}{\partial \log \mu_R^2} + \mathcal{O}(\alpha_s^{k+3})$$

- σ soll nicht von μ_R abhängen:

$$0 = \alpha_s^{k+1} \left(-4\pi k \beta_0 \sigma_0 + \frac{\partial \sigma_1(\mu_R)}{\partial \log \mu_R^2} \right)$$

Renormierungsskalenabhängigkeit

- σ soll nicht von μ_R abhängen:

$$0 = \alpha_s^{k+1}\left(-4\pi k\beta_0\sigma_0 + \frac{\partial\sigma_1(\mu_R)}{\partial \log \mu_R^2}\right)$$

- Auflösen nach σ_1:

$$\begin{aligned}\sigma_1(\mu_R) &= -4\pi k\beta_0\sigma_0 \log \mu_R^2 + \tilde{\sigma}_1 \\ &= -4\pi k\beta_0\sigma_0 \log \frac{\mu_R^2}{\mu_F^2} + \tilde{\sigma}_1(\mu_F^2)\end{aligned}$$

- μ_R Abhängigkeit der NLO-Korrektur ist proportional zu σ_0
- Integrationskonstante ist μ_R-unabhängiger Teil
  (aber noch $\sim \mu_F$)

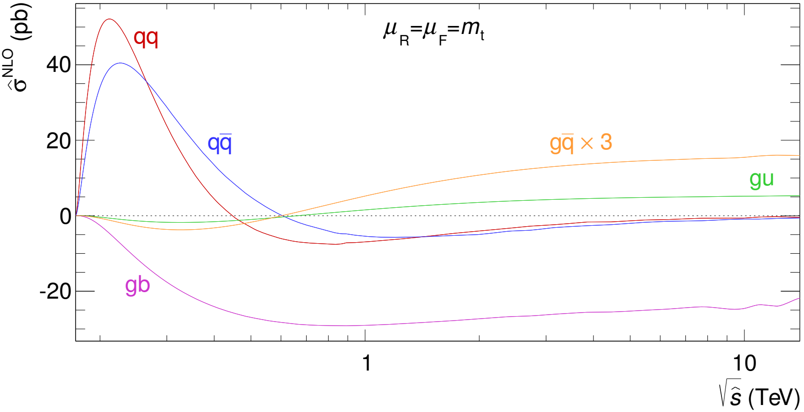
- DGLAP-Gleichung ($2n_f + 1$ Dimensionen):

$$\frac{\partial}{\partial \log \mu_{\mathrm{R}}^2} \begin{pmatrix} q_i(x, \mu_{\mathrm{F}}) \\ g(x, \mu_{\mathrm{F}}) \end{pmatrix} = 4\pi\alpha_{\mathrm{s}} \sum_{q_i, \bar{q}_j} \begin{pmatrix} P_{q_i, q_j} & P_{q_i, g} \\ P_{g, q_j} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_i \\ g \end{pmatrix} (x)$$

- Entwickle P_{ij} in Potenzen von α_{s}
- Gleichungen müssen für alle PDF gelten
- Verlange für beide Skalen

$$\frac{\partial \sigma}{\partial \log \mu^2} = 0$$

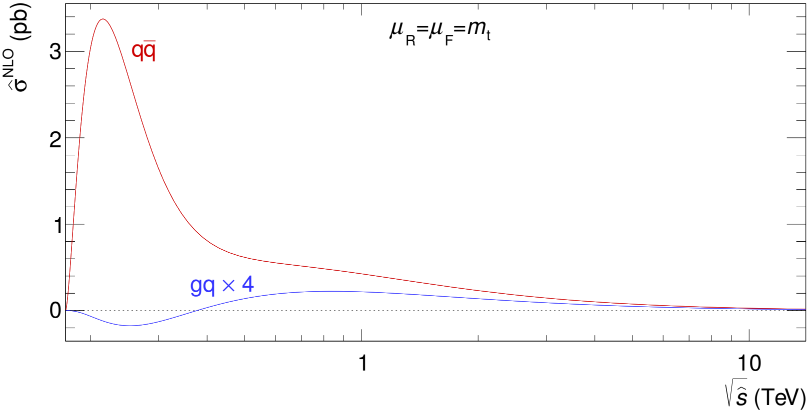
NLO-Beiträge im *t*-Kanal



$$\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{LO} + \alpha_s^{k+1} \, \hat{\sigma}^{NLO} + \ldots$$

Partonische Wirkungsquerschnitte

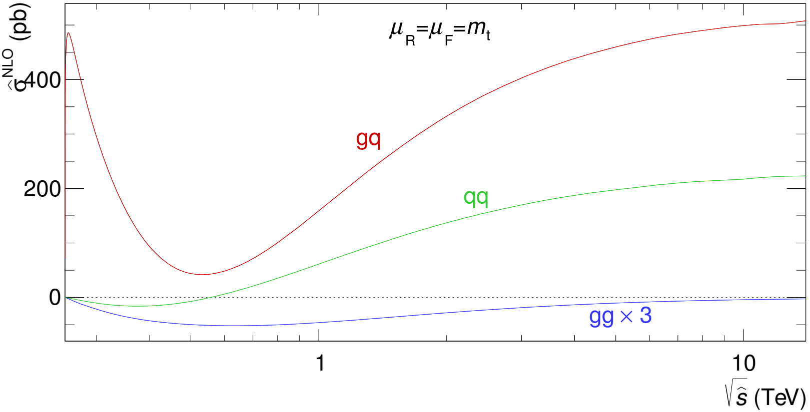
NLO-Beiträge im *s*-Kanal



$$\hat{\sigma} = \alpha_s^k\,\hat{\sigma}^{LO} + \alpha_s^{k+1}\,\hat{\sigma}^{NLO} + \ldots$$

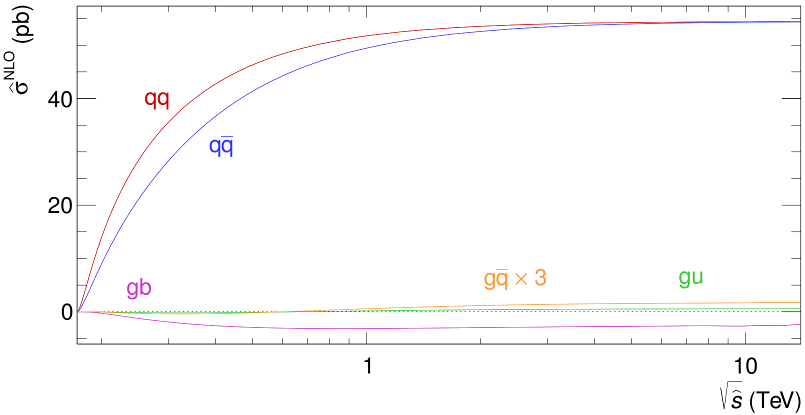
Partonische Wirkungsquerschnitte

NLO-Beiträge zur Wt-Produktion



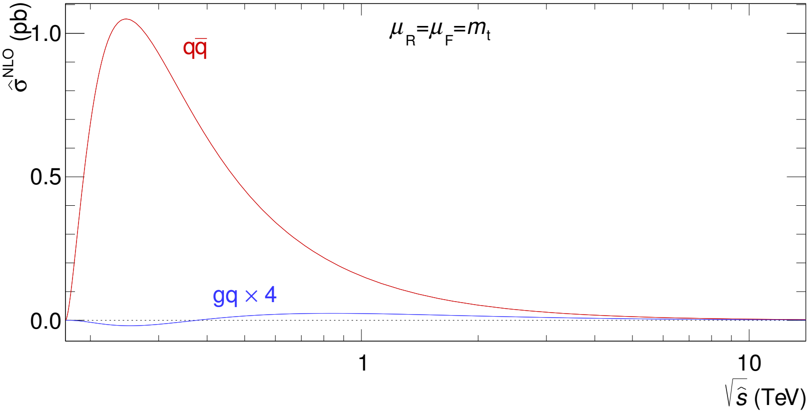
$$\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{LO} + \alpha_s^{k+1} \, \hat{\sigma}^{NLO} + \dots$$

LO+NLO-Beiträge im *t*-Kanal



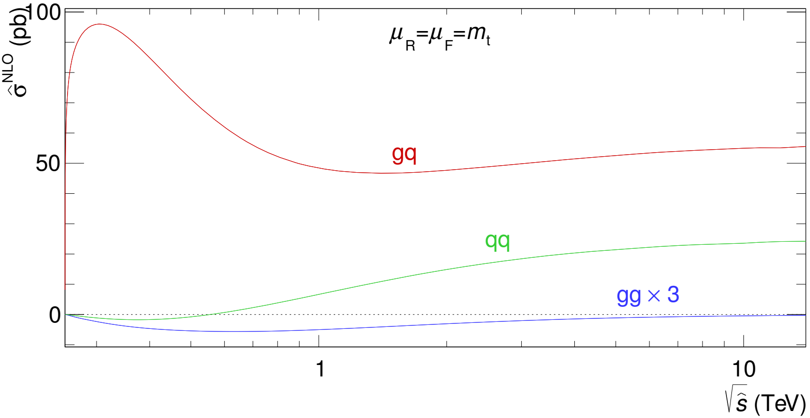
$$\hat{\sigma} = \alpha_s^k\, \hat{\sigma}^{LO} + \alpha_s^{k+1}\, \hat{\sigma}^{NLO} + \ldots$$

LO+NLO-Beiträge im *s*-Kanal



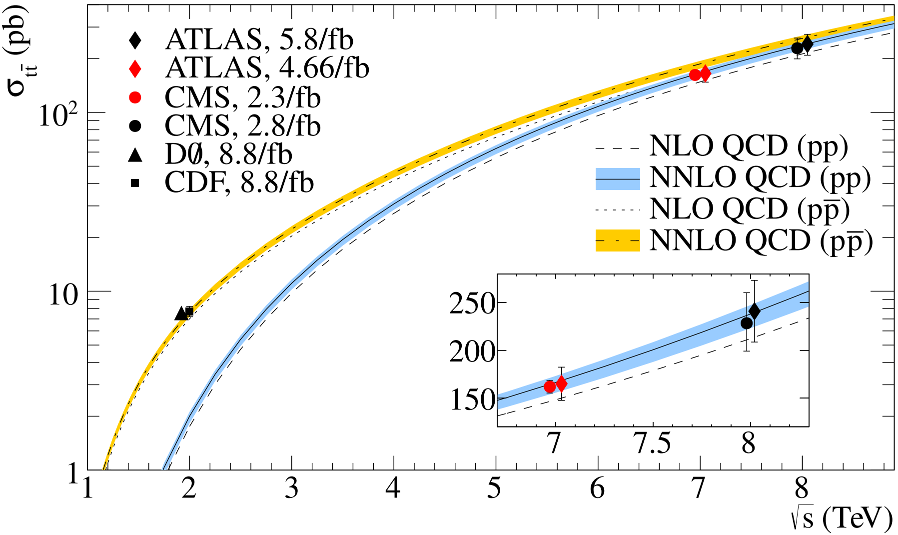
$$\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{LO} + \alpha_s^{k+1} \, \hat{\sigma}^{NLO} + \ldots$$

LO+NLO-Beiträge zur Wt-Produktion

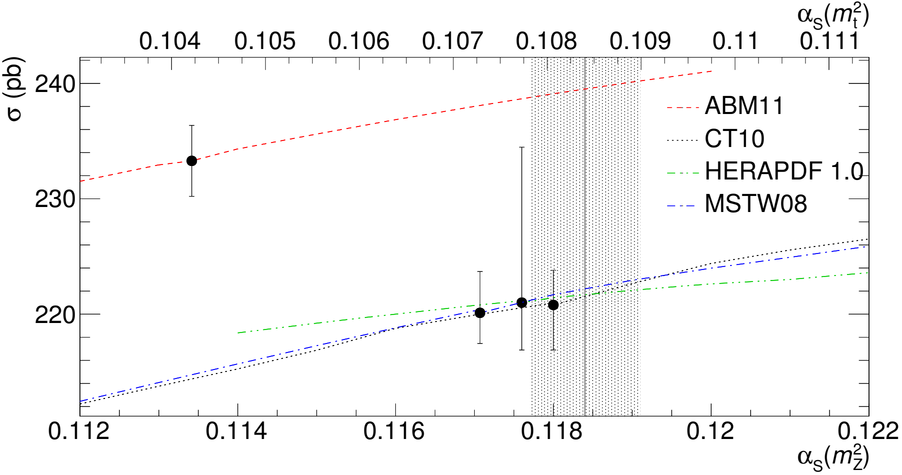


$$\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{\mathrm{LO}} + \alpha_s^{k+1} \, \hat{\sigma}^{\mathrm{NLO}} + \ldots$$

HATHOR für Top-Paarerzeugung

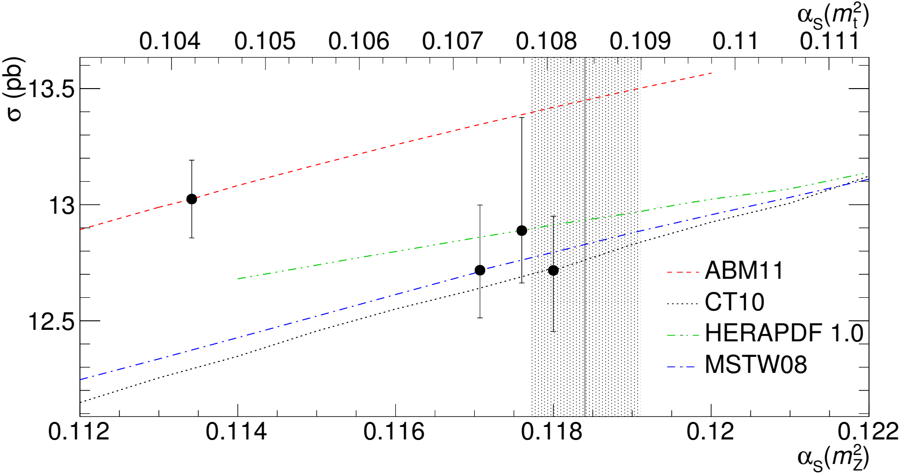


α_s-Abhängigkeit im t-Kanal



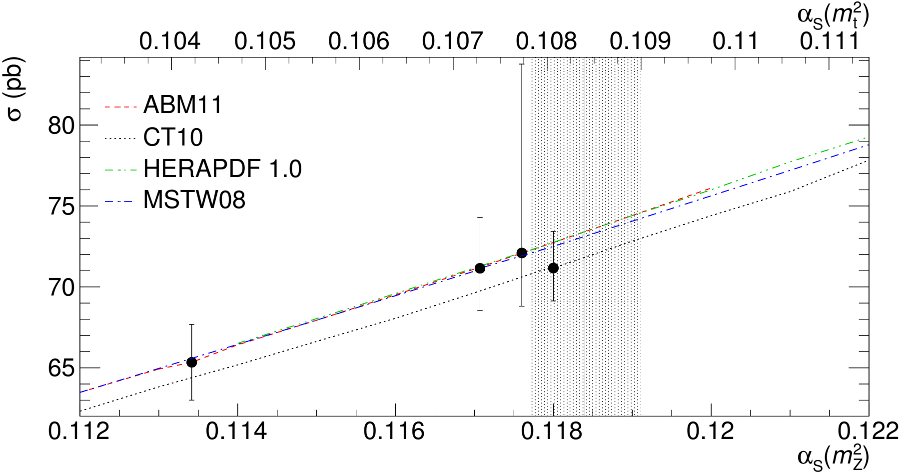
(in approx. NNLO, $\sqrt{s} = 14$ TeV)

α_s-Abhängigkeit im s-Kanal



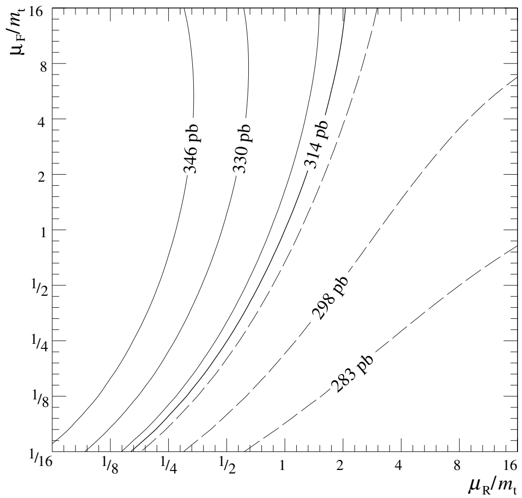
(in approx. NNLO, $\sqrt{s} = 14$ TeV)

α_s-Abhängigkeit der Wt-Produktion

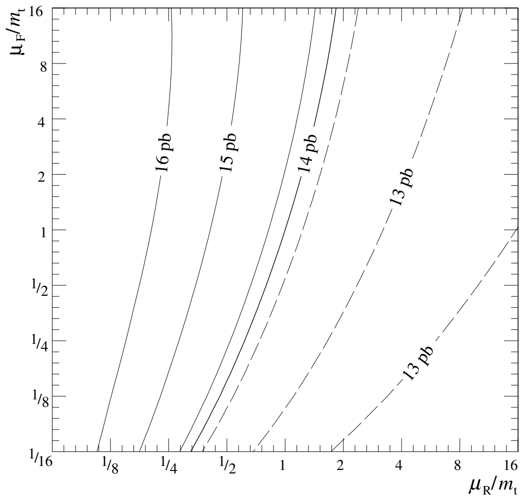


(in approx. NNLO, $\sqrt{s} = 14\,\text{TeV}$)

Skalenabhängigkeit im t-Kanal

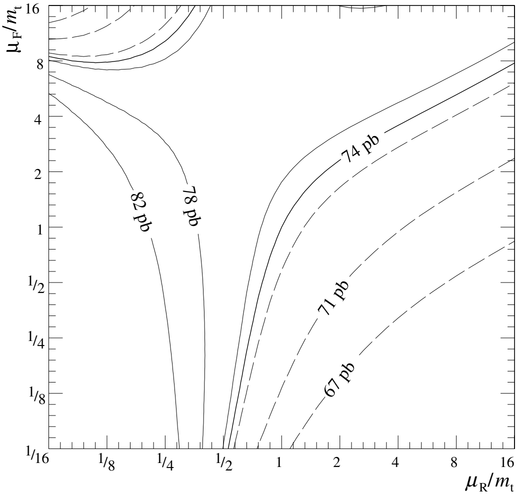


(in approx. NNLO, $\sqrt{s} = 14$ TeV)



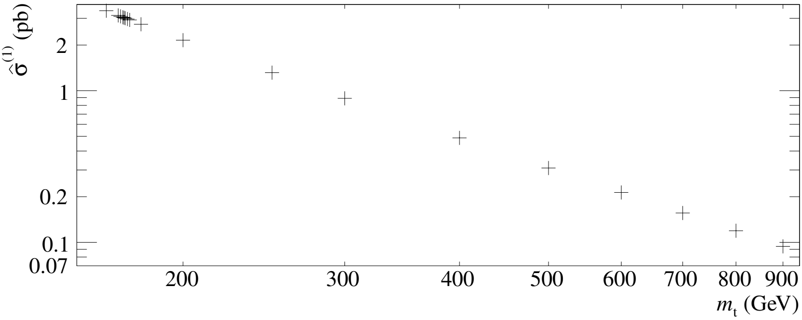
(in approx. NNLO, $\sqrt{s} = 14$ TeV)

Skalenabhängigkeit der Wt-Produktion

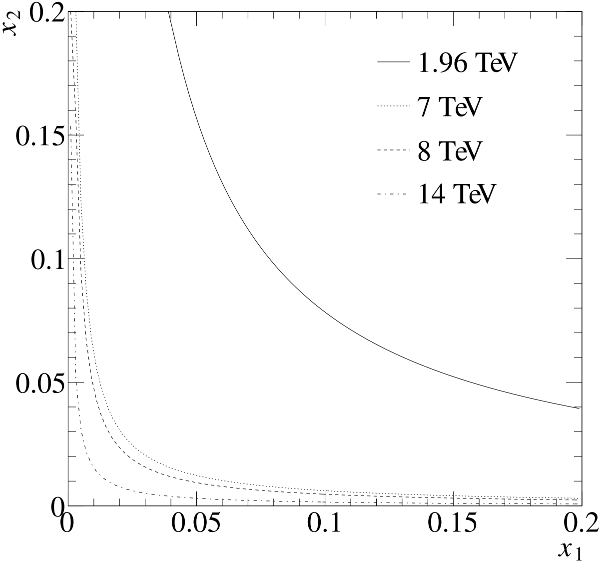


(in approx. NNLO, $\sqrt{s} = 14$ TeV)

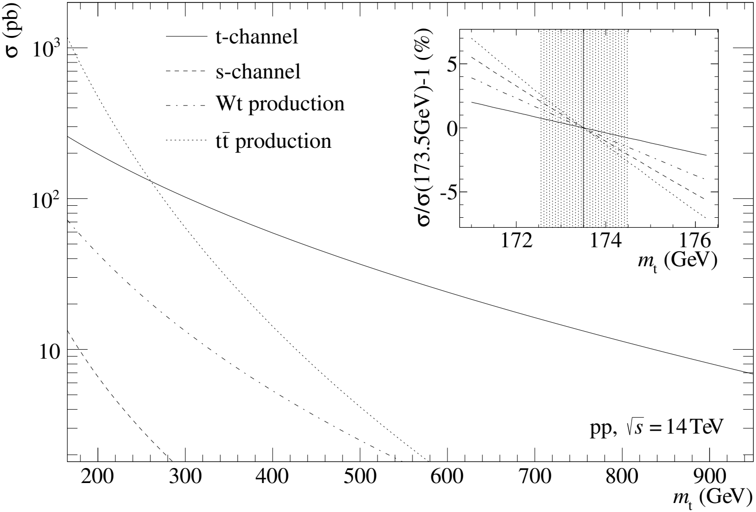
Massenabhängigkeit (part. WQ)



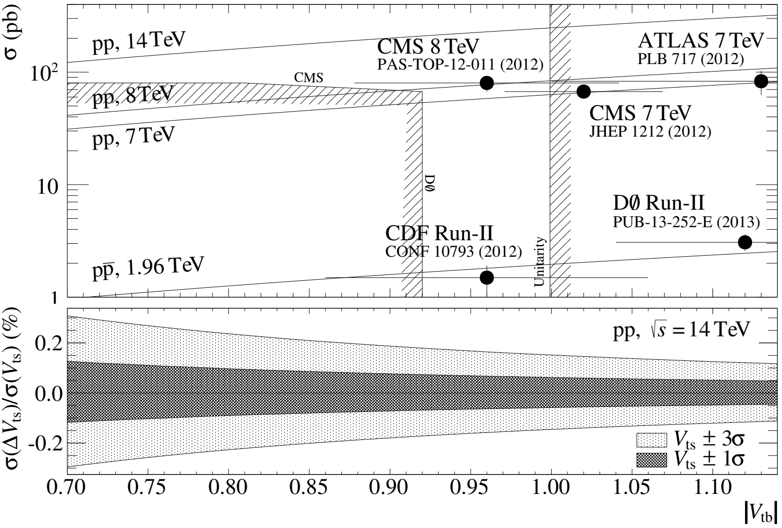
Zugänglicher Bereich in $x_{1,2}$



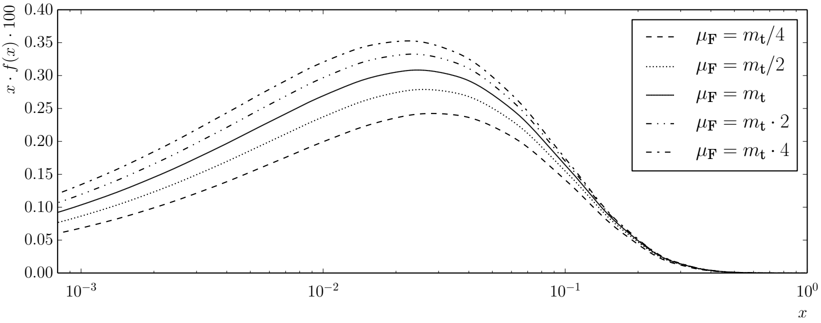
Massenabhängigkeit



CKM-Matrixelementabhängigkeit

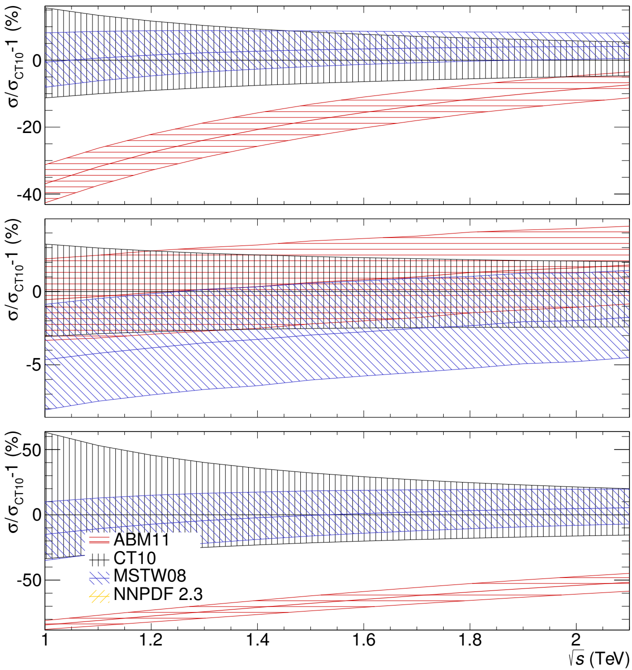


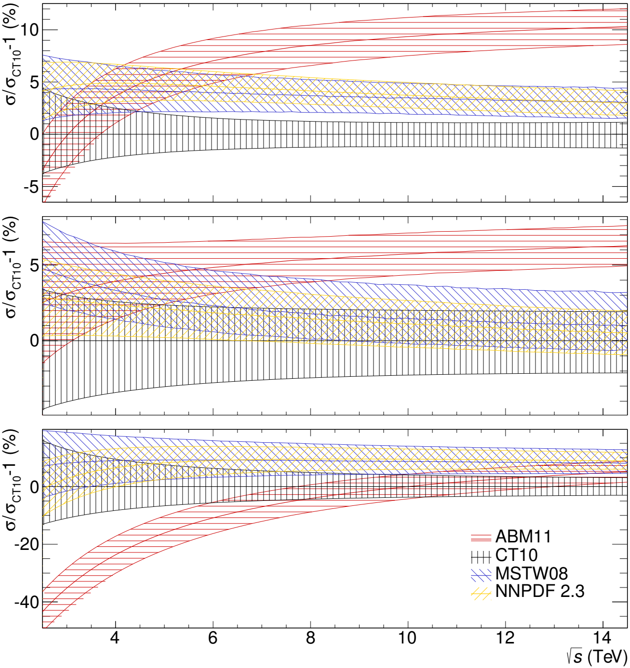
Bottom-PDF



0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$x \cdot f(x) \cdot 100$
10^{-3}
10^{-2}
10^{-1}
10^{0}
x
$\mu_F = m_t/4$
$\mu_F = m_t/2$
$\mu_F = m_t$
$\mu_F = m_t \cdot 2$
$\mu_F = m_t \cdot 4$

PDFs (Tevatron)





NLO-PDF (aNNLO $\hat{\sigma}$)