Mass in Particle Physics
Mass and CKM Mixing

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Quark Mixing: The CKM Matrix

- Effect of the basis transformation:
  - Mass matrices become diagonal
  - Interaction with $\mathrm{Re}\,\phi_0$ (= Physical Higgs Boson) becomes diagonal !
  - Interaction with $\mathrm{Im}\,\phi_0$ (= Z_0) becomes diagonal !

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Re}\,\phi_0} &= -\mathrm{Re}\,\phi_0[\mathcal{U}_L Y^u \mathcal{U}_R + \mathcal{D}_L Y^d \mathcal{D}_R] \\
\mathcal{L}_{\mathrm{Im}\,\phi_0} &= -\mathrm{Im}\,\phi_0[\mathcal{U}_L Y^u \mathcal{U}_R - \mathcal{D}_L Y^d \mathcal{D}_R]
\end{aligned}
$$

- NO FLAVOUR CHANGING NEUTRAL CURRENTS (at tree level in the Standard Model)
- $\rightarrow$ GIM Mechanism

- Effect on the charged current ONLY: Interaction with ϕ_-:

$$\sum_{ij} \bar{Q}_i (y_i \delta_{ij} + y'_{ij}) \phi_- \tau_- P_+ q_j + \text{ h.c.}$$
$$= \mathcal{D}_L Y^u \mathcal{U}_R \phi_- + \text{ h.c.}$$
$$= \bar{\mathcal{D}}_L U^{d,\dagger} (U^d U^{u,\dagger}) Y^u_{diag} W^u \mathcal{U}_R \phi_- + \text{ h.c.}$$

- In the charged currents flavour mixing occurs!
- Parametrized through the Cabbibo-Kobayashi-Maskawa Matrix:

$$V_{CKM} = U^d U^{u,\dagger}$$

Properties of the CKM Matrix

- V_{CKM} is unitary (by our construction)
- Number of parameters for n families
  - Unitary $n \times n$ matrix: n^2 real parameters
  - Freedom to rephase the $2n$ quark fields: $2n - 1$ relative phases
- $n^2 - 2n + 1 = (n-1)^2$ real parameters
  - * $(n-1)(n-2)/2$ are phases
  - * $n(n-1)/2$ are angles
- Phases are sources of *CP* violation
- $n = 2$: One angle, no phase $\to$ no *CP* violation
- $n = 3$: Three angles, one phase
- $n = 4$: Six angles, three phases

CKM Basics

- Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix} , \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- Single phase δ: $\quad U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} .$

- PDG CKM Parametrization:

$$V_{\mathrm{CKM}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

- Large Phases in $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$ and $V_{td} = |V_{td}| e^{i\beta}$

CKM Unitarity Relations

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Off diagonal zeros of $V_{CKM}^\dagger V_{CKM} = 1 = V_{CKM} V_{CKM}^\dagger$
- $V_{CKM}^\dagger V_{CKM} = 1 : \begin{cases} V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0 \\ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0 \\ V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^* = 0 \end{cases}$
- $V_{CKM} V_{CKM}^\dagger = 1 : \begin{cases} V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \\ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \\ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \end{cases}$

Wolfenstein Parametrization of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

- Expansion in $\lambda \approx 0.22$ up to λ^3
- A, ρ, η of order unity

Unitarity Triangle(s)

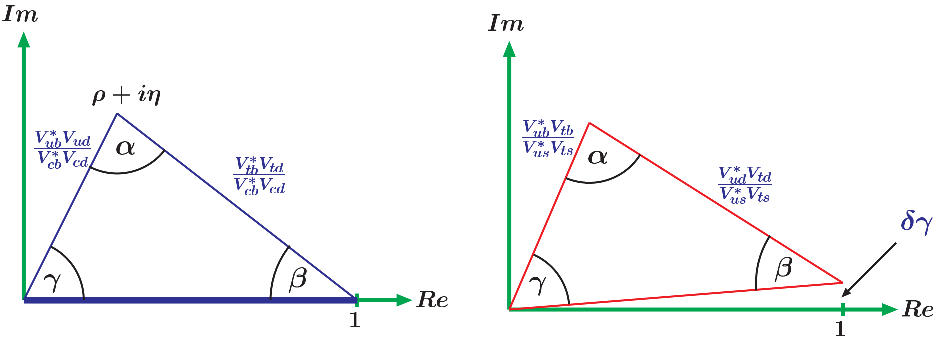
- The unitarity relations:
  Sum of three complex numbers = 0
- Triangles in the complex plane
- Only two out of the six unitarity relations involve terms of the same order in λ:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

- Both correspond to

$$A\lambda^3(\rho + i\eta - 1 + 1 - \rho - i\eta) = 0$$

- This is THE unitarity triangle ...



- Definition of the CKM angles α, β and γ
- To leading order Wolfenstein:

$$V_{ub} = |V_{ub}| e^{-i\gamma} \qquad V_{tb} = |V_{tb}| e^{-i\beta}$$

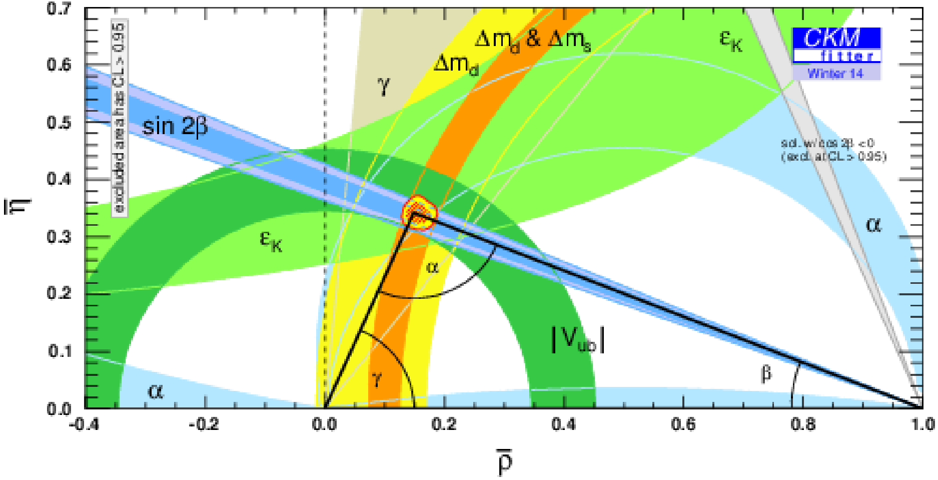
  all other CKM matrix elements are real.
- $\delta\gamma$ is order λ^5

- Aerea of the Triangle(s): Measure of CP Violation
- Invariant measure of CP violation:

$$\mathrm{Im}\Delta = \mathrm{Im}\, V_{ud} V_{td}^* V_{tb} V_{ub}^* = c_{12} s_{12} c_{13}^2 s_{13} s_{23} c_{23} \sin\delta_{13}$$

- Maximal possible value $\delta_{\max} = \frac{1}{6\sqrt{3}} \sim 0.1$
- CP Violation is a small effect:
  Measured value $\delta_{\exp} \sim 0.0001$
- CP Violation vanishes in case of degeneracies: (Jarlskog)

$$\begin{aligned} J &= \mathrm{Det}([M_u\,,\, M_d]) \\ &= 2i\mathrm{Im}\Delta(m_u - m_c)(m_u - m_t)(m_c - m_t) \\ &\quad \times (m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned}$$


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
$\bar{\eta}$
$\bar{\rho}$
excluded area has CL > 0.95
sin 2β
γ
Δm_d & Δm_s
Δm_d
ε_K
ε_K
CKM fitter
Winter 14
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
α
α
γ
β
$|V_{ub}|$