Mass in Particle Physics
Neutrino Masses

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Schule für Astroteilchenphysik 2014

Contents

Introduction

Leptons still have an additional aspect of “mass”

- A possible right-handed neutrino does not have any quantum numbers in the SM
  - it carries no charge
  - it does not couple to $W^{\pm}$ and Z
  - it only would come in to generate a (Dirac) mass
- this allow for another type of mass term: A Majorana mass

Leptons in the Standard Model

- If the neutrinos are massless:
  - Only left handed neutrinos couple
  - No flavor mixing in the lepton sector
- Recent evidence for neutrino mixing:
  - This requires a mass term
  - Mixing in the Lepton Sector
- It could be just a copy of the quark sector, but it may be different due to the quantum numbers of the leptons

Multiplets and Quantum Numbers

- Left Handed Leptons: $SU(2)_L$ Doublets

$$L_1 = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \; L_2 = \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \; L_3 = \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

- Right Handed Leptons: $SU(2)_R$ Doublets

$$\ell_1 = \begin{pmatrix} \nu_{e,R} \\ e_R \end{pmatrix} \; \ell_2 = \begin{pmatrix} \nu_{\mu,R} \\ \mu_R \end{pmatrix} \; \ell_3 = \begin{pmatrix} \nu_{\tau,R} \\ \tau_R \end{pmatrix}$$

- Charge and Hypercharge

$$Y = T_{3,R} + \frac{1}{2}(B - L) = T_{3,R} - \frac{1}{2} \qquad q = T_{3,L} + Y$$

- Y (and q) project the lower component: Right handed Neutrinos: No charge, no Hypercharge

Majorana Fermions

- A “neutral” fermion can have a Majorana mass
- Charged fermions $\Leftrightarrow$ complex scalar fields
- Majorana fermion: “Real (= neutral) fermion”
- Definition of “complex conjugation” in this case:
  Charge Conjugation:

$$\psi \to \psi^c = C\bar{\psi}^T \quad C = i\gamma_2\gamma_0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}$$

- Properties of C

$$-C = C^{-1} = C^T = C^\dagger$$

- Majorana fermion: $\psi_{Majorana} = \psi^c_{Majorana}$
  (Just as $\phi^* = \phi$ for a real scalar field)

Majorana Mass Terms

- Mass term for a Majorana fermion: The charge conjugate of a right handed fermion is left handed.
- Possible mass term

$$\mathcal{L}_{MM} = -\frac{1}{2} M \left(\bar{\nu}_R (\nu_R^c)_L + h.c. \right)$$

- Only for fields without $U(1)$ quantum numbers
- In the SM: only for the right handed neutrinos !
- Remarks:
  - The Majorana mass of the right handed neutrinos is NOT due to the Higgs mechanism.
  - Thus this majorana mass can be "large"
  - Natural explanation of the small neutrino masses: see-saw mechanism

See Saw Mechanism

- Simplification: One family: ν_L and ν_R
- Total Mass term: Dirac and Majorana mass

$$\mathcal{L}_{mass} = -m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) - \frac{1}{2} M(\nu_R^T C \nu_R + \bar{\nu}_R C \bar{\nu}_R^T)$$

- We use

$$\overline{(\nu_R^c)}_L (\nu_L^c)_R = \bar{\nu}_L \bar{\nu}_R$$

and the properties of the C matrix ...

$$\mathcal{L}_{mass} = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \overline{(\nu_R^c)}_L \end{pmatrix} \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} (\nu_L^c)_R \\ \nu_R \end{pmatrix} + h.c.$$

- Diagonalization of the mass matrix:
  $\rightarrow$ Majorana mass eigenstates of the Neutrinos
  For $M \gg m$ we get

$$m_1 \approx \frac{m^2}{M} \quad m_2 \approx M$$

- One very heavy, practically right handed neutrino
- One very light, practically left handed neutrino
- At energies small compared to M:
  Majorana mass term for the left handed neutrino

$$\mathcal{L}_{mass} = -\frac{1}{2}\frac{m^2}{M}\left(\nu_L^T C \nu_L + \bar{\nu}_L C \bar{\nu}_L{}^T\right)$$

- Majorana mass is small if $M \gg m$

Right handed neutrinos in the Standard Model

- In case of three families: **Neutrino Mixing**
- Compact notation for the Leptons:

$$\mathcal{N}_{L/R} = \begin{bmatrix} \nu_{e,L/R} \\ \nu_{\mu,L/R} \\ \nu_{\tau,L/R} \end{bmatrix} \qquad \mathcal{E}_{L/R} = \begin{bmatrix} e_{L/R} \\ \mu_{L/R} \\ \tau_{L/R} \end{bmatrix}$$

- Dirac masses are generated by the Higgs mechanism: (as for the quarks)

$$\mathcal{L}_{DM}^{N} = -\mathcal{N}_L m^N \mathcal{N}_R + h.c.$$
$$\mathcal{L}_{DM}^{E} = -\mathcal{E}_L m^E \mathcal{E}_R + h.c.$$

- m^N: Dirac mass matrix for the neutrinos
- m^E: (Dirac) mass matrix for e, μ, τ

- Right handed neutrinos $\to$ Majorana mass term:

$$\mathcal{L}_{MM} = -\frac{1}{2}\left(N_R^T M C N_R + \bar{N}_R M C \bar{N}_R^T\right)$$

- M: (Symmetric) Majorana Mass Matrix
- This term is perfectly $SU(2)_L \otimes U(1)$ invariant
- Implementation of the see saw mechanism:
  Assume that all Eigenvalues of M are large
- Effective Theory at low energies:
  Only light, practically left handed neutrinos
- Effect of right handed neutrino:
  Majorana mass term for the light neutrinos

$$\mathcal{L}_{mass} = -\frac{1}{2}\left(N_L^T m^T M^{-1} m C N_L + \bar{N}_L m^T M^{-1} m C \bar{N}_L^T\right)$$

In case you do not like right handed neutrinos:
"Effective Theory Picture of new physics"

- Add higher dimensional operators:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda_{\rm BSM}} \sum_k C_k^{(5)} \mathcal{O}_k^{(5)} + \frac{1}{\Lambda_{\rm BSM}^2} \sum_k C_k^{(6)} \mathcal{O}_k^{(6)} + \cdots$$

- Only a single type of dim-5 operator in the SM

$$\mathcal{O}_{ij}^{(5)} = (L_i H^c)^c \left(H^{\dagger,c} L_j\right)$$

with H: Higgs field and the left-handed lepton doublet

$$L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}$$

- Upon symmetry breaking, this operator generates (majorana) neutrino masse term $\mathcal{L}_{mass}$

Lepton Mixing: PMNS Matrix

- Diagonalization of the Mass matrices:
  - Charged leptons:

$$m^E = U^\dagger m^E_{diag} W$$

  - Neutrinos: “Orthogonal” transformation:

$$m^T M^{-1} m = O^T m^\nu_{diag} O \text{ with } O^\dagger O = 1$$

- Again no Effect on neutral currents
- Charged Currents: Interaction with ϕ_+:

$$\frac{1}{v} \mathcal{N}_L m^E \mathcal{E}_R \phi_+ + \text{ h.c.}$$
$$= \frac{1}{v} \bar{\mathcal{N}}_L O^T (O^* U^\dagger) m^E_{diag} W \mathcal{E}_R \phi_+ + \text{ h.c.}$$

- A Mixing Matrix occurs:

$$V_{PMNS} = O^* U^\dagger$$

Pontecorvo Maki Nakagawa Sakata Matrix

- V_{PMNS} is unitary like the CKM Matrix
- Left handed neutrinos are Majorana: No freedom to rephase these fields!
    - For n families: n^2 Parameters
    - Only n Relative phases free
    - $\longrightarrow n(n-1)$ Parameters
    - $n(n-1)/2$ are angles
    - $n(n-1)/2$ are phases: More sources for CP violation

- Almost like CKM: Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix} \quad , \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- A Dirac Phase δ and two Majorana Phases α_1 and α_2

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} \quad U_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{bmatrix}$$

- PMNS Parametrization: $V_{\mathrm{PMNS}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12} U_\alpha$
- $\Theta_{23} \sim 45^\circ$ is “maximal” (atmospheric ν’s)
- $\Theta_{13} \sim 0$ is small (ν’s from reaktors)
- $\sin\Theta_{13} \sim 1/\sqrt{3}$ is large (solar ν’s)

Maltoni et al '04

parameter	best fit	2σ	3σ	5σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	≤ 0.035	≤ 0.054	≤ 0.11

$$V_{\mathrm{PMNS}} \sim \begin{bmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \end{bmatrix} \sim \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{bmatrix}$$

- No Hierarchy !

Consequences of Lepton Mixing

- FCNC Processes in the leptonic Sector:

$$\tau \to \mu\gamma \quad \mu \to e\gamma \quad \tau \to eee \text{ etc.}$$
$$\nu_\tau \to \nu_e\gamma \quad \nu_\tau - \nu_e \text{ mixing}$$

- Lepton Number Violation:
  Right handed Neutrinos are Majorana fermions:
  No conserved quantum number corresponding to the rephasing of the right handed neutrino fields
  Lepton number violation could feed via conserved $B - L$ into Baryon number violation
  Relation to the Baryon Asymmetry of the Universe ?

Summary of Lecture 3

- Majorana Mass terms can appear for right handed neutrinos
- ... which are NOT related to the Higgs coupling
- There is no reason why this mass term could not be as large as the GUT scale
- ... which would explain in turn the small (observed) neutrino mass (differences)
- A Majorana mass term induces Lepton Number Violation
  Key experiment is the neutrino-less double β decay

Many open Questions ...

Masses and Mixings leave many open questions:

- What is the origin of the three(?) families?
- Why are the (fundamental) masses so different?
- If they are really generated by the Higgs mechanism: Why are the Yukawa couplings so small?
- Why are the "fundamental" mass scales so different?

$$\Lambda_{\text{cosm. const.}} \ll \langle \bar{q}q \rangle^{1/3} \ll v \ll M_{\text{Planck}}$$

- Messages from Gravity / Cosmology:
  - Is there really dark matter, and (if yes) what ist it?
  - What is "dark energy"?

Overall Summary

- The phenomenon “mass” has many different aspects
- Probably one of the mort important aspects is
- ... Mass gravitates
- Expect further clues form cosmological findings (such as dark matter, dark energy etc.)

Maybe in the years form now,
this lecture would look completely different