Mass in Particle Physics
QCD and the Mass of Hadrons

Thomas Mannel

Theoretische Physik I, Universität Siegen

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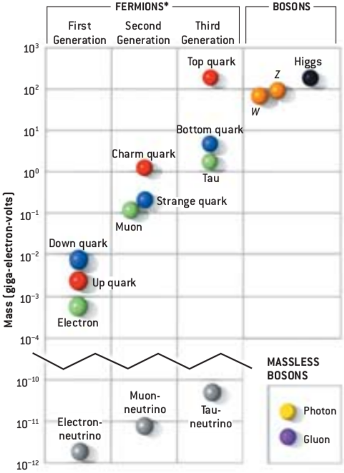
Introduction / Motivation

Masses of the fundamental fermions



- The proton consists of up and down quarks
- We have [PDG]

$$m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV} \quad \text{and} \quad m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$$

$$\boxed{\text{How can the proton mass be 938 MeV?}}$$

- Almost the full mass must be “binding energy”
- Certainly not a loosely-bound system
- The mass of the quarks seems to be negligible, so ...

$$\boxed{\text{How can the proton mass for vanishing quark masses?}}$$

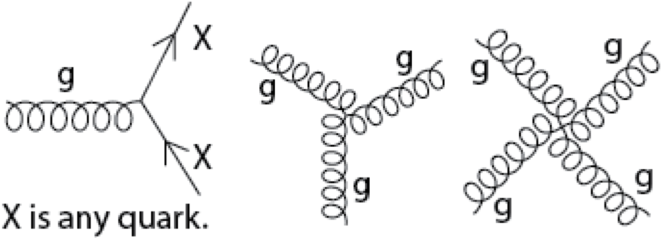
Quantum Field Theory in a Nutshell

- We have considered classical field theory for the SM
- QCD is part of the SM:

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{q=u,d,s} \bar{q} i \not{D} q - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a}$$

  where we look only at the light quarks $q = u, d, s$ and the gluons $F^a_{\mu\nu}$
- There is no dimensional quantity in this Lagrangian
- ... so how can a mass emerge?

- (Perturbative) Quantum Field Theory
- Feynman Rules



- Feynman Digramms: Define your external lines and draw all possible diagrams with a given number of vertices
- This translates into a mathematical expression for the quantum-mechanical amplitude.

However, some of the expressions are infinite!

- The "loop-momentum" can be infinitely high
- Integrand does not cut-off the high momentum modes
- These contributions need "renormalization"
- Renormalization is a general feature of QFT

The Essence of Renormalization

- Physical parameters (masses and couplings) are obtained form subtracting a “counter term contribution” from the “bare parameter”
- Renormalization requires to fix the parameters of the theory by experimental input.
- Example: The QCD coupling is determined by performing a scattering experiment at a high scale μ

$$\sigma(\mu) \sim \frac{\alpha_s(\mu)}{\mu^2}$$

- This defines a “running coupling”
- Perturbative expansion in $\alpha_s(\mu)$
- This always introduces a “renormalization scale” μ

Running Coupling

- The running is defined by the β function

$$\mu \frac{d}{d\mu} \alpha_s(\mu) = \beta(\alpha_s(\mu))$$

- the β function is (perturbatively) calculated from the diagrams

- Result known up to four loops

- One loop result

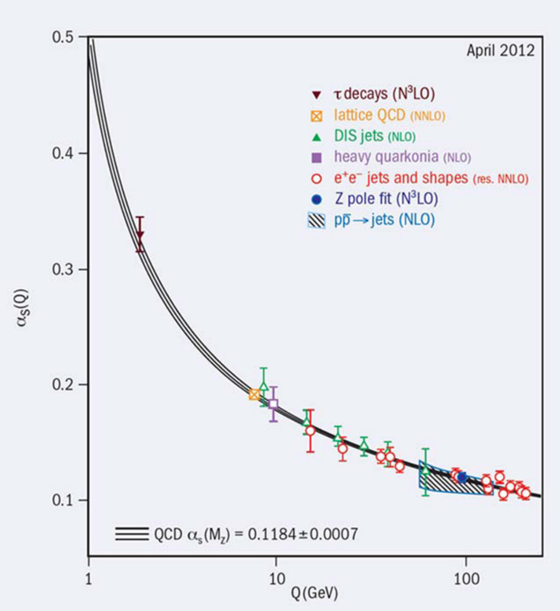
$$\beta(\alpha_s) = \left(\frac{2n_f}{3} - 11\right)\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3) = \beta_0 \alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

- General solution for the running coupling

$$\frac{d\mu}{\mu} = \frac{d\alpha_s}{\beta(\alpha_s)} \qquad \ln\left(\frac{\mu}{\mu_0}\right) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}$$

- One loop running

$$\beta_0 \ln\left(\frac{\mu}{\mu_0}\right) = \frac{1}{\alpha_s(\mu_0)} - \frac{1}{\alpha_s(\mu)} \qquad \alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 - \beta_0 \ln\left(\frac{\mu}{\mu_0}\right)}$$


April 2012
τ decays (N³LO)
lattice QCD (NNLO)
DIS jets (NLO)
heavy quarkonia (NLO)
e⁺e⁻ jets and shapes (res. NNLO)
Z pole fit (N³LO)
pp̄ → jets (NLO)
QCD $\alpha_s(M_Z) = 0.1184 \pm 0.0007$
$\alpha_s(Q)$
0.5
0.4
0.3
0.2
0.1
1
10
100
Q (GeV)

Dimensional Analysis for Pedestrians

Consider a dimensionless quantity R (such as $s\sigma(s)$)

and pretend for a moment there were no renormalization !

- Perturbative expansion in QCD

$$R = R(\alpha_s) = \sum_{n=0}^{\infty} r_n \alpha_s^n$$

- In massless QCD (no scale!) this cannot depend on any momentum transfer q^2

$$R(q^2) \to R\left(\frac{q^2}{\Lambda^2}\right), \qquad r_n \text{ and } \alpha_s \text{ dimensionless}$$

but there is no Λ in (finite) QCD.

- If everything would be finite, naive dimensional analysis would hold
- Conclusion: at sufficiently high scales $\sigma(s) \sim 1/s$
- However: α_s is running, i.e. it depends on a scale
- „, but which is arbitrary

How can we get something from nothing?

Mass from dimensional Transmutation

- The renormalization scale is still arbitrary
  Nothing can depend on it
- How to get a "renormalization invariant" mass scale?
- Construct $\Lambda_{\rm QCD}(\mu, \alpha_s(\mu))$

$$\Lambda_{\rm QCD}(\mu, \alpha_s(\mu)) = \mu \exp\left(- \int\limits_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \right)$$

- Show that

$$\mu \frac{d}{d\mu} \Lambda_{\rm QCD}(\mu, \alpha_s(\mu)) = 0 = \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) \Lambda_{\rm QCD}(\mu, \alpha_s)$$

- One loop result

$$\Lambda_{\rm QCD}(\mu, \alpha_s) = \text{const}\, \mu \exp\left(-\frac{1}{\beta_0 \alpha_s}\right)$$

  (the constant is usually put to unity)
- Value of $\Lambda_{\rm QCD}$ depends on the scheme in which renormalziation is performed
- Usual scheme is “modified minimal subtraction” $\overline{\rm MS}$

$$\Lambda_{\rm QCD, \overline{\rm MS}} = (340 \pm 8)\ {\rm MeV}$$

  (for three quark flavors)
- QCD generates a mass scale of a few hundred MeV!

A few remarks:

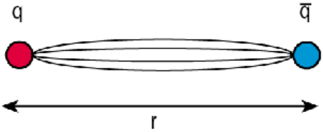
- Renormalizable theories always generates their mass scale, even if you start out massless!
- this looks like something from nothing, but there is the input of running α_s
- This scale is nonperturbative: There is no Taylor series of $\exp(1/x)$
- The dimensional analysis has to be modified due to the presence of this scale!

- Trivial example: Coulomb Potential
- Maxwell Theory is scaleless, no renormalization
- Potential as a function of r between static charges: Dimensional analysis: Energy $\sim$ inverse length, thus

$$V(r) = \frac{\text{const}}{r}$$

- Pure-Glue QCD: scaleless, but needs renorm.:
- Potential between two static color charges:

$$V(r) = \frac{1}{r}\, f\left(\frac{r}{\Lambda_{\mathrm{QCD}}}\right) \to \sigma\, r$$



Intermediate Summary

- Massless QCD generates a mass scale due to renormalization
- Even in massless QCD we thus expect to have a non-vanishing proton mass!
- Explains the mismatch between the light quark masses (emerging from the Higgs mechanism) and the large proton mass,

More on Masses in QCD: Chiral Symmetry

- Massless QCD (for the three light quarks) has an $U(3)_L \times U(3)_R$ Symmetry

$$\mathcal{L} = \sum_k \bar{q}_{k,L} i \not{D} q_{k,L} + \sum_k \bar{q}_{k,R} i \not{D} q_{k,R} + \text{gluons}$$

- However, that symmetry is not seen in nature, only the diagonal symmetry $SU(2)_{L+R}$ is actually seen.
- „, corresponding to Isospoin or Flavour $SU(3)$
- If the symmetry were exact: no proton mass
- ASSUMPTION: $SU(3)_L \times SU(3)_R \to SU(2)_{L+R}$ is a spontaneous symmetry breaking (SSB)

- This SSB generates Goldstone particles: Pions and Kaons
- The "order parameter" is the quark condensate $\langle \bar{q} q \rangle = \langle 0 | \bar{q} q | 0 \rangle$ (corresponding to the Higgs VEV)
- ... this couples left an right handed components
- Pions and Kaons get (small) masses from explicit symmetry breaking from the quark mass terms:

$$\mathcal{L}_{\rm mass} = \bar{q}_L \mathcal{M} q_R + \text{h.c.} \qquad \mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

- Relation for the pion and kaon mass

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2}(m_u + m_d) \qquad m_K^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2}(m + m_s),$$

  with $m = \frac{1}{2}(m_u + m_d)$
- From this we get

$$\langle \bar{q}q \rangle \sim (-0.23 \text{ GeV})^3$$

Back to massless QCD:

- Also the quark condensate has to originate from dimensional transmutation

$$\langle \bar{q} q \rangle = \text{const.} \times \Lambda_{\rm QCD}^3$$

  with a constant of order unity
- This is a consistent picture, however, difficult to quantify

Finally: The mass of the proton

- Start from the proton state at rest: $|p\rangle$
- This should be an eigenstate of the hamiltonian:

$$H|p\rangle = \int d^3\vec{x}\, T^{00}(x)\, |p\rangle = M_{\mathrm{Proton}}|p\rangle$$

  with the energy-momentum tensor $T^{\mu\nu}$
- From this we get

$$\langle p|\, T^{00}(0)\, |p\rangle = M_{\mathrm{Proton}}$$

- In massless QCD T^{00} is scaleless, so the mass can only come form dimensional transmutation

- The proton mass does not vanish in the chiral limit, thus it is not proportional to the quark masses
- approximate relation (Ioffe)

$$M_{\mathrm{Proton}}^3 \sim 2(2\pi)^3 \langle \bar{q} q \rangle$$

which actually yields $M_{\mathrm{Proton}} \sim 1$ GeV.

Summary on Lecture 2

- The proton mass (and all other hadron masses, except for the loins an kaons) has NOTHING to do with the Higgs
- Even in the massless limit, QCD is believed to have a non-trivial mass spectrum
- The mechanism is dimensional transmutation, which generates a scale
- The hadron mass spectrum is related to the breaking of the chiral symmetry of QCD