MASS in Particle Physics

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Overview

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  - Basics of the Standard Model
  - Spontaneous Symmetry Breaking
  - The Higgs Mechanism
- Lecture 2: QCD and the Mass of Hadrons
  - Quantum Field Theory in a Nutshell
  - Dimensional Analysis for Pedestrians
  - Mass from “Dimensional Transmutation”

- Lecture 3: Neutrino Masses
  - Majorana Masses for Fermions
  - Dirac and Majorana Masses
  - See-Saw Mechanism and small Neutrono masses
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Mass in Particle Physics
Basics and the Standard Model

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Introduction

- Particle Physicists describe the world in terms of (Quantum) Field Theory.
- Observed particles are “field quanta”

  (like the photon is the field quantum of Maxwell’s Field Theory)
- Similarly to Classical mechanics, (systems of) fields are described by Lagrangians
- Classical mechanics: Generalized Coordinates q_i

$$L(q_i, \dot{q}_i) : \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = 0$$

- Field theory: Fields $\phi_i(x)$, and “Lagrangian Densities”

$$\mathcal{L} = \mathcal{L}(\phi_i(x), \partial_\mu \phi_i(x))$$

- Equations of motion:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) - \left(\frac{\partial \mathcal{L}}{\partial(\phi_i)} \right) = 0$$

- Example (and Exercise): Maxwell's Equations

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + j_\mu A^\mu \qquad \text{yields} \qquad [\Box A_\mu - \partial_\mu(\partial_\nu A^\nu)] = j_\mu$$

- A quadratic Lagrangian always yields a linear equation of motion, corresponding to a “free field”
- Quadratic terms:

$$(\partial_\mu \phi_i)(\partial^\mu \phi_i) \qquad \phi_i \phi_i \qquad (\partial_\mu \phi_i)\phi_i$$

- The last one is not allowed: not Lorentz Invariant!

- Most general quadratic term

$$\mathcal{L} = a\,(\partial_\mu \phi_i)(\partial^\mu \phi_i) - b\,\phi_i \phi_i \tag{1}$$

- Remark on dimensions:
  Particle Physicists units: $\hbar = 1$, $c = 1$, $k_b = 1$
  Energy and Mass have the same unit
  Length is an inverse mass
  Temperature and Energy have the same unit
- Unit of Action (= interal over the Lagrangian)

$$S = \int d^4x\, \mathcal{L}(\phi_i(x), \partial_\mu \phi_i(x))$$

is $\hbar$ thus Dimensionless

- We may chose the unit of ϕ such that a is dimension-less: $\dim[\phi_i] = 1$
  (= the unit of ϕ_i is "mass")
- The dimension of b is $\dim[b] = 2$
  (= the unit of b is "mass-squared")
- Once the field ϕ_i with the lagrangian (1) is quantized, its quanta correspond to particles with mass $m^2 = b$, assuming that b is positive!
  Otherwise ... (see later)
- The field quanta satisfy $E = \sqrt{m^2 + \vec{p}^2}$

Mass in a QFT:

Quadratic Term in the fields with no derivatives

Mass terms for the different particle species

- For scalar particles:

$$\mathcal{L}_{\rm mass} = \frac{1}{2} m^2 \phi^2 \quad \text{or} \qquad \mathcal{L}_{\rm mass} = m^2 \phi^* \phi$$

- For vector particles

$$\mathcal{L}_{\rm mass} = -\frac{1}{2} m^2 A_\mu A^\mu \quad \text{or} \qquad \mathcal{L}_{\rm mass} = -m^2 A_\mu^* A^\mu$$

- Spin 1/2 particles: four component Spinors ψ
- Free spin 1/2 particles: Dirac Equation:

$$(i\gamma_\mu \partial^\mu - m)\psi(x) = (i\not\partial - m)\psi(x) = 0$$

with $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}$

- Corresponding Lagrangian Density

$$\mathcal{L}_{1/2} = \bar{\psi}(x)(i\not\partial - m)\psi(x) \qquad \bar{\psi}(x) = \psi^\dagger(x)\gamma_0$$

- The mass term is thus

$$\mathcal{L}_{\text{mass}} = -m\bar{\psi}(x)\psi(x)$$

- Spinors can be classified by their Chirality:
  Matrix γ_5 with $\gamma_5\gamma_5 = 1$ and $\gamma_\mu\gamma_5 + \gamma_5\gamma_\mu = 0$

$$\psi_L(x) = \frac{1}{2}(1-\gamma_5)\psi(x) \qquad \text{Left-handed Component}$$
$$\psi_R(x) = \frac{1}{2}(1+\gamma_5)\psi(x) \qquad \text{Right-handed Component}$$

- Note that

$$\bar{\psi}_L(x) = \frac{1}{2}\bar{\psi}(x)(1+\gamma_5)$$
$$\bar{\psi}_R(x) = \frac{1}{2}\bar{\psi}(x)(1-\gamma_5)$$

Note that $\bar{\gamma}_5 = -\gamma_5$

- Show that

$$\mathcal{L}_{\rm mass} = -m\,[\bar{\psi}_L(x)\psi_R(x) + \bar{\psi}_R(x)\psi_L(x)]$$

- Mass term couples the left and right handed components
- There can also be a Majorana Mass for Spinors (see Lecture 3)

The Standard Model of Particle Physics

Properties

- (Lagragian) Quantum Field Theory
- Construction based on Symmetries
- "Chiral" Theory: Left and right handed components of spin 1/2 particles are treated differently.
- "Gauge field Theory" local symmetries, enforcing massless vector bosons
- Masses are generated by "spontaneous symmetry breaking"

Basics Structure

- Symmetries:
  - $SU(3)$ for color (not really relevant here)
  - $SU(2)$ for weak interactions
  - $U(1)$ for "electromagnetic" interactions
- Quarks and Leptons fall into "weak doublets" (under to SU(2))
- For the left handed component of the quarks

$$Q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \; Q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \; Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

- Right handed components of the quarks: singlets

- For the left handed component of the leptons

$$L_1 = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \; L_2 = \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \; L_3 = \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

- No need for right handed neutrinos, since we assume them massless (See lecture 3)
- Right handed charged leptons are singlets.
- Local gauge symmetry: Vector Bosons massless!
    - Only two polarization directions
    - A massive vector boson has three polarizations
    - ... thus we miss one degree of freedom
    - Higgs Mechanism: A massless scalar particle can be "eaten up" and become the longitudinal polarization of a massive vector boson

Scalar Particles

Consider a charged scalar particle = complex scalar field

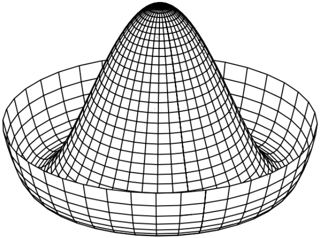
$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi^* \phi)$$

- This is invariant under phase rotations: $\phi \to \phi e^{i\alpha}$ and thus $\phi^* \to \phi^* e^{-i\alpha}$: $\mathcal{L} \to \mathcal{L}$
- The potential of a renormalizale QFT must be

$$V(\phi^* \phi) = \alpha \, (\phi^* \phi) + \beta \, (\phi^* \phi)^2$$

- The first term looks like a mass term, but ...

- .. only if $\alpha = m^2 > 0$
- β has to be positive, since the potential has to be bounded form below.
- For $\alpha < 0$ we get a "mexican hat" potential:



- In such a case we get "spontaneous symmetry breaking"

To see how this happens,
lets chose a specific parametriation

$$\phi(x) = \frac{1}{\sqrt{2}} \rho(x) \exp\left(\frac{i}{v}\eta(x)\right)$$

with η and ρ real fields. We get

- $\phi^*\phi = \frac{1}{2}\rho^2$ and thus $V(\phi^*\phi) = V\left(\frac{1}{2}\rho^2\right)$
- The potential does not depend on η
- $\partial_\mu\phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{v}\eta\right) \left[\partial_\mu\rho + \rho\frac{i}{v}\partial_\mu\eta\right]$
  and thus
  $(\partial_\mu\phi)^*(\partial^\mu\phi) = \frac{1}{2}\left[(\partial_\mu\rho)(\partial^\mu\rho) + \frac{\rho^2}{v^2}(\partial_\mu\eta)(\partial^\mu\eta)\right]$

We need to expand around the “ground state”:
Need to minimize the potential

- This implies for the “ground state configuration” ϕ_0 a non-vanishing value $(\phi_0^* \phi_0) = \frac{1}{2} v^2$
- This is implemented by

$$\phi(x) = \frac{1}{\sqrt{2}} (\rho(x) + v) \exp\left(\frac{i}{v}\eta(x)\right)$$

$$\phi_0 = \frac{1}{\sqrt{2}} v$$

- As an exercise, calculate v in terms of α and β; prove that α has to be negative in order to have SSB

- Insert this into the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)(\partial^\mu \rho) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - V\left(\frac{1}{2}\rho^2\right)$$
$$+\text{interaction terms between } \rho \text{ and } \eta$$

- The ρ becomes massive (as an exercise, calculate its mass in term of α and β)
- The η is a massless field: Goldstone mode.
- The interactions between ρ and η are independent of the potential

Toy Model for Fermion Masses

- “Chiral” fermions:

$$\mathcal{L} = \bar{\psi}_L i \partial\!\!\!/ \psi_L + \bar{\psi}_R i \partial\!\!\!/ \psi_R$$

- Symmetry: Independent phase transformations for left and right handed components

$$\psi_L \to \psi_L e^{i\alpha} \quad \psi_R \to \psi_R e^{i\beta}$$

- This forbids a mass term of the from $\bar{\psi}_L \psi_R$

$$\bar{\psi}_L \psi_R \to \bar{\psi}_L \psi_R e^{-i(\alpha-\beta)}$$

- However, one can introduce a Higgs field and use SSB

- Introduce a complex scalar field ϕ with the transformation

$$\phi \to \phi e^{i(\alpha-\beta)}$$

- Now we can write an invariant term (Yukawa Coupling)

$$\mathcal{L}_{\text{int}} = \lambda \phi \bar{\psi}_L \psi_R + \text{h.c.}$$

  λ: Yukawa coupling constant

- ... but once the symmetry gets spontaneously broken, we get

$$\mathcal{L}_{\text{int}} = \frac{\lambda}{2} v \, \bar{\psi}_L \psi_R + \text{h.c.} + \cdots$$

- ... which is a mass term with a mass value proportional to v and the Yukawa coupling

How do the Vector Bosons become massive?

Look first at electrodynamics ...

- Lagrangian for Maxwell Equation:

$$\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Add a complex scalar field and couple it "minimally gauge invariant" to the photon

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - V(\phi^* \phi)$$

with $D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$

- Invariance under local phase transformations

$$\phi \to \phi \exp[ie\,\alpha(x)] \qquad A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

- Use the parametrization (including already SSB)

$$\phi(x) = \frac{1}{\sqrt{2}}(\rho(x) + v) \exp\left(\frac{i}{v}\eta(x)\right)$$

The field η can be removed by a clever choice

$$\alpha = \frac{1}{ev}\eta$$

The field η can be "gauged away"

- What happens to this degree of freedom?

- Look at the rest of the Lagrangian in the gauge, where η is gone (Unitary gauge!)

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi = \frac{v}{\sqrt{2}} ieA_\mu + \cdots$$

and thus

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v^2 e^2 A_\mu A^\mu + \cdots$$

- The Vector Boson became massive
- ... and thus has a longitudinal polarization
- The η field has disappeared
  (“eaten” by the Vector boson)

The mass terms of the full Standardmodel

In principle, you know now all the ingredients:

- Explicit mass terms for the fermions are forbidden, since we treat left and right handed components differently (Doublets versus Singlets)
- Explicit mass terms for the Vector Bosons are forbidden by local $SU(2) \times U(1)$ Symmetry
- Elegant (and renormalziable) solution: Spontaneous symmetry breaking
- Introduce a (complex) $SU(2)$ doublet H

$$H = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad H^c = \begin{pmatrix} \phi_0^* \\ -\phi_+^* \end{pmatrix}$$

with appropriate hypercharge assignments

- Chose a (renormalzable) potential such that

$$H_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad H_0^c = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

- Write $SU(2) \times U(1)$ invariant Yukawa couplings

$$\mathcal{L} = y_d(\bar{Q}_L H) d_R + y_u(\bar{Q}_L H^c) u_R + \text{h.c.}$$

- Upon SSB: mass terms of up and down quarks
- The Higgs doublet has four degrees of freedom Decomposes into one massive mode and three massless Goldstone modes
- The three massless modes get eaten: $W^{\pm}$ and Z_0 become massive.
- Photon (and gluons) remain massless

Three generations

The is a slight complication: We have three generations!

- y_d and y_u: 3×3 matrices in generation space
- Upon SSB, this results in mass matrices
- A physical quark is a mass eigenstate!
- Look at the mass matrices

$$M_u = \frac{v}{2} Y_u \qquad M_d = \frac{v}{2} Y_d$$

- There is no reason what both matrices should be diagonal in the same basis

$$\left[M_u M_u^\dagger \, , \; M_d M_d^\dagger \right] \neq 0$$

- The relative rotation between the two eigenbases:s Cabbibo, Kobayashi Maskawa (CKM) matrix

Some remarks on masses and the CKM matrix

- CKM is the source of the mixing of quark flavors
  Flavour mixing is linked to the masses
- The CKM matrix also encodes the CP violation present in the SM
- Masses and Mixing parameters have quite peculiar values
  - The values of the masses span an enormous range
  - This corresponds to very small Yukawa couplings
    Except for the top quark, which has a "normal" coupling
  - The quark flavor mixing also follows a hierarchy
    Again small numbers that have to be explained
- Overall, we only seem to have a successful parametrization, but no understanding

Summary on Lecture 1

- All masses of the fundamental fermions are generated by SSB
  (possible exception: Neutrinos)
- Fermion masses are Yukawa couplings $\times$ v
- Vector Boson Masses are gauge coupling $e \times v$
- Quark Flavor Mixing is related to the mass terms
- We have a successful parameterization, but not really an understanding
- Even worse: Most of the mass around us is NOT due to what I jet told you
  stay tuned for Lecture 2