Loop Quantum Gravity – Part I: Introduction

Kristina Giesel

NORDITA

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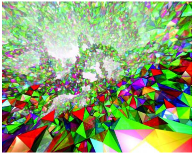

Plan of the Talk

- Part I:
  - Motivation for a theory of Quantum Gravity
  - Candidate: Loop Quantum Gravity
  - Classical starting point: General Relativity
  - General Relativity & background independence
  - Canonical quantisation of GR
  - Quantum – Einstein – Equations and their interpretation
- Part II:
  - Current research project within Loop Quantum Gravity
  - Geometrical operators (volume, area & length)
  - Solutions to Quantum – Einstein – Equations
- Summary & Conclusions

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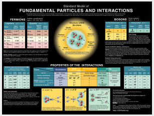
Modern Theoretical Physics

- Two foundations: QFT and GR



Standard Model: Quantum Field Theory

- Describes fundamental interaction of electromagnetic, weak and strong force
- Description uses perturbative techniques (Exception: Lattice Gauge Theory)
- Infinities occur in perturbation series
- Renormalisation techniques can eliminate these infinities

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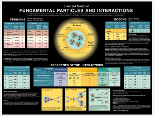

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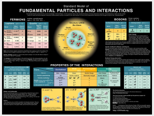

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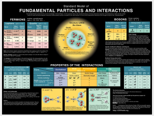

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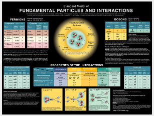

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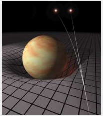

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- Describes fundamental interaction between matter & space time geometry
- Interaction is encoded in Einstein's equations
- Big Bang in cosmology, black holes $\longrightarrow$ singularities
- Singularity theorems in General Relativity

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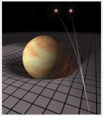

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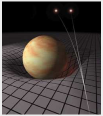

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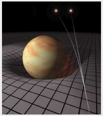

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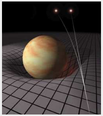

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- Both show singularities $\longrightarrow$ validity

Requirements for a Theory of Quantum Gravity:

- Adequate description of Quantum – Geometry & Quantum – Matter
- Principles of GR & QFT should be implemented consistently
- Background independence & canonical quantisation

Quest for a Theory of Quantum Gravity

Hope: Quantum Gravity is Fundamental Theory

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Classical Einstein's – Equations

Interaction Between Space Time Geometry & Matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad = \quad T_{\mu\nu}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\text{Geometry} \quad = \quad \text{Matter}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\text{Classical} \quad ? \quad \text{Quantum}$$

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General Relativity

Basics of General Relativity

- General Covariance: Physics does not depend on coordinates

Einstein's Equations

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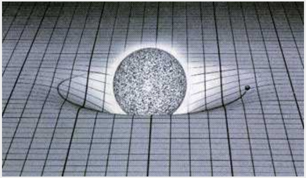

- Metric $g_{\mu\nu}$ encodes space time geometry
- NO preferred background metric, metric becomes dynamical object
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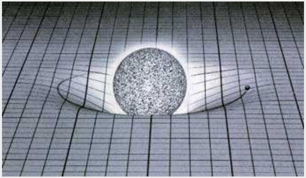

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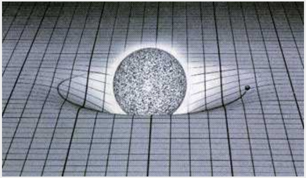

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Metric

Metric encodes geometry of spacetime

- In particle physics we assume Minkowski spacetime

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Friedmann - Robertson - Walker - Metric (FRW)

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a(t) & 0 & 0 \\ 0 & 0 & a(t) & 0 \\ 0 & 0 & 0 & a(t) \end{pmatrix}$$

- Line element: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- $ds^2_M = -dt^2 + dx^2 + dy^2 + dz^2$, $ds^2_{FRW} = -dt^2 + a(t)(dx^2 + dy^2 + dz^2)$

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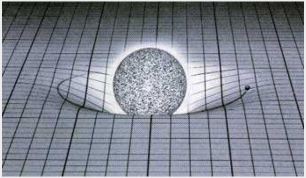

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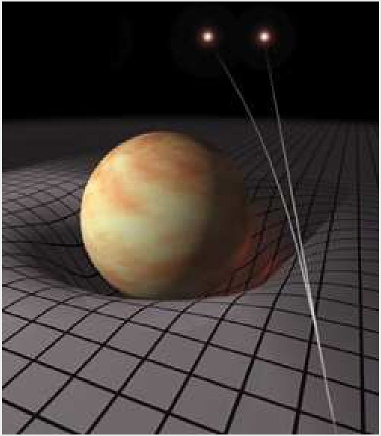
Background Independence

Geometry and thus metric becomes dynamical object



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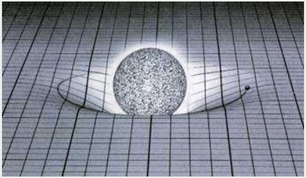

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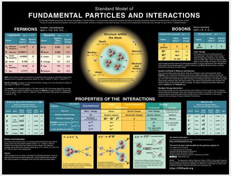
Quantum Field Theory

Basics of Quantum Field Theory

- QFT combines classical Field Theories & Quantum Mechanics

Canonical Quantisation in QM

$$\{q_j, p^k\} = \delta_j^k \longrightarrow [\hat{q}_j, \hat{p}^k] = i\hbar\delta_j^k$$



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- Wightman Axioms rely on high symmetry of Minkowski-metric
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- Quantum theory: Canonical quantisation (Path integrals)

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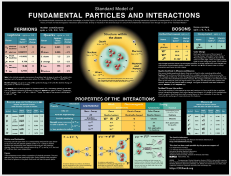

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Canonical Quantisation

Canonical Quantisation: Quantum Mechanics

- Elementary phase space variables q_i, p^j satisfy Poisson algebra $\{q_i, p^j\} = \delta_i^j$
- Dynamics described by Hamiltonian $H(p, q)$, example harmonic oscill. $H(p, q) = \frac{p^2}{2m} + \frac{1}{2}m\omega q^2$
- Construct a corresponding quantum algebra which respects the usual canonical quantisation rule $[\hat{q}, \hat{p}] = i\hbar$
- Look for possible representations of this quantum algebra
- Operators $\hat{q}\psi(x) := x\psi(x)$, $\hat{p}\psi(x) := -i\hbar\frac{d}{dx}\psi(x)$
- Stone – von Neumann – theorem for QM: Schrödinger representation unique

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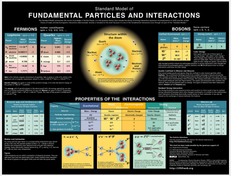

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- Elementary phase space variables $\phi(\mathrm{x})$,$\pi(x')$ satisfy $\{\phi(\mathrm{x}),\pi(\mathrm{x}')\} = \delta(\mathrm{x},\mathrm{x}')$
- Smear $\phi(\mathrm{x})$ and $\pi(\mathrm{x}')$ with test functions
- $\phi(\mathrm{f}) := \int \mathrm{d}^3\mathrm{x}\phi(\mathrm{x})\mathrm{f}(\mathrm{x})$, $\pi(\mathrm{f}) = \int \mathrm{d}^3\mathrm{x}\pi(\mathrm{x})\mathrm{f}(\mathrm{x})$
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Basics of Quantum Field Theory

- QFT combines classical Field Theories & Quantum Mechanics

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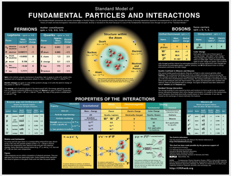

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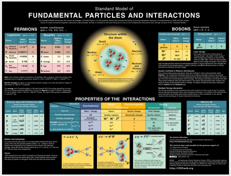

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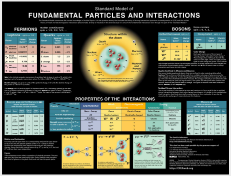

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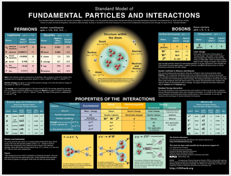

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Request for Theory of Quantum Gravity

Are we only bothered by occurence of infinites?

- Anisotropies in cosmic microwave background
- Cosmological constant problem
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Loop Quantum Gravity

Loop Quantum Gravity: Candidate for a Quantum Theory of GR

Loop Quantum Gravity

1. Step:

- Derive Hamiltonian formulation of General Relativity

2. Step:

- Apply canonical quantisation to Hamiltonian form of General Relativity

3. Step:

- Quantum analogue of classical Einstein's Equations: Quantum – Einstein – Equations

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Lagrange: (Vacuum) Einstein's Equations

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Hamiltonian Formulation (ADM – formalisms)

- Split into space & time: $\int_{\mathcal{M}} \to \int_{\mathbb{R}} \int_{\sigma}$
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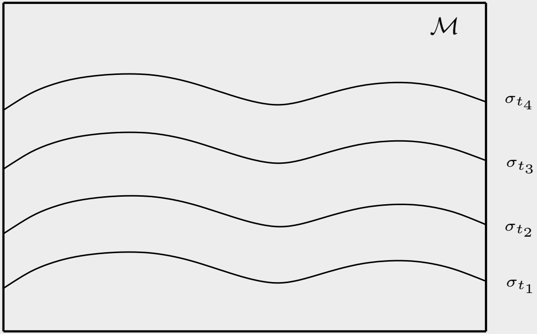
Foliation of Space Time

(3+1) Split into Space & Time



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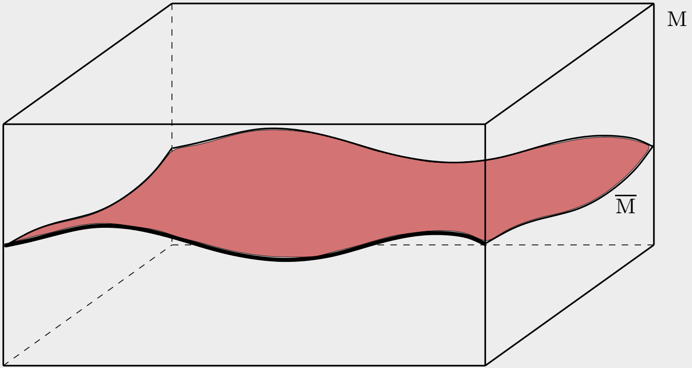
Classical Formulation of GR

Constraint Hypersurface



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Langrangian Framework

- Einstein equations: $R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = 0$
- $g_{\mu\nu}$ has 10 independent components $\longrightarrow$ 10 equations
- Only six of them are second order differential equations

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Constraints of General Relativity

Explicit Form in ADM Variables for Vacuum Case:

- Diffeomorphism constraint

$$D_a(q,p) = -2q_{ac}p^{bc}_{;b}$$

- Hamiltonian constraint

$$H(q,p) = \frac{1}{\sqrt{\det(q)}}\left(q_{ac}q_{bd} - \frac{1}{2}q_{ab}q_{cd}\right)p^{ab}p^{cd} - \sqrt{\det(q)}R(q)$$

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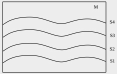
Hamiltonian Formulation [Ashtekar '86]



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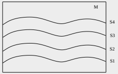
Classical Formulation of GR

Lagrange: (Vacuum) Einstein's Equations

$$S = \int_{\mathcal{M}} d^4x \sqrt{|\det(g)|}R \qquad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0$$

Hamiltonian Formulation [Ashtekar '86]



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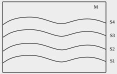

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Constraints of General Relativity

Explicit Form in Ashtekar – Variables for Vacuum Case:

- Gauß constraint

$$\mathrm{G_j(A,E) = D_a E^a_j}$$

- Diffeomorphism constraint

$$\mathrm{D_a(A,E) = F^j{}_{ab} E^b_j} \qquad F^j_{ab} \text{ curvature of } A^j_a$$

- Hamiltonian constraint

$$\mathrm{H(A,E) = \frac{1}{\sqrt{\det(E)}} \left(\epsilon_i^{jk} F^i{}_{ab} E^a_j E^b_k \right)}$$

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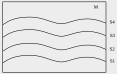

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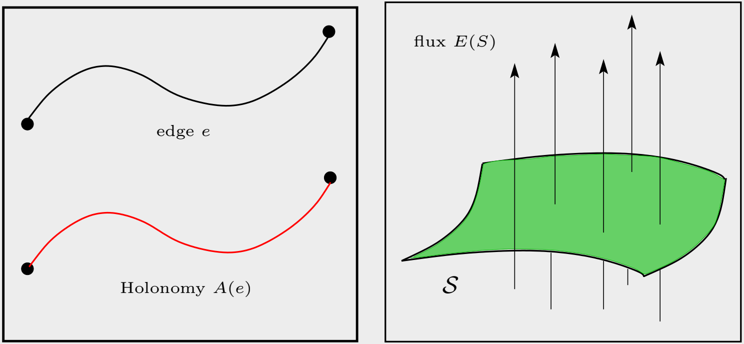
Elementary phase space variables of LQG

Holonomies A(e) and Fluxes E(S)



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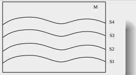

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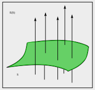

Canonical Quantisation for Constrained Systems

Option 1: Dirac's Programme for Constraint Quantisation

- Quantise kinematical theory $\{\mathrm{q_{ab}}(\mathrm{x}), \mathrm{p^{cd}}(\mathrm{x}')\} = \delta^3(\mathrm{x}, \mathrm{x}')\delta_\mathrm{a}^\mathrm{c}\delta_\mathrm{d}^\mathrm{b}$
- Solve constraints in Quantum theory
  one gets a kinematical Hilbert space $\mathcal{H}_{\mathrm{kin}}$
- Physical states are then $\widehat{\mathrm{C}}\psi_{\mathrm{phys}} = 0$
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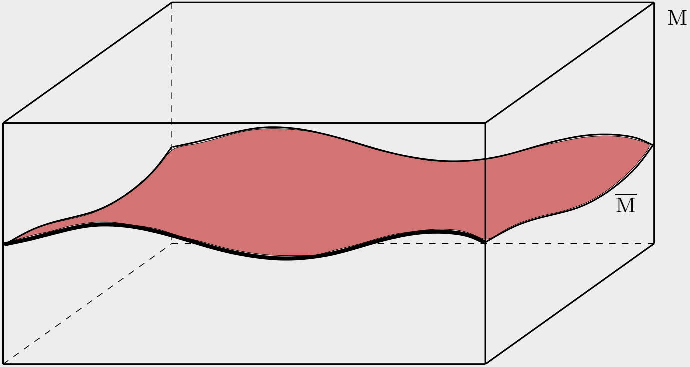
Two Options for Constrained Systems

Option 1 & 2: Quantising M Or Only $\overline{\mathrm{M}}$



Quantisation for Loop Quantum Gravity

Option 1 or Option 2?

- It does not mean that one of the options is preferred
- Different strategies to quantise systems with constraints
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Recall from Quantum Mechanics

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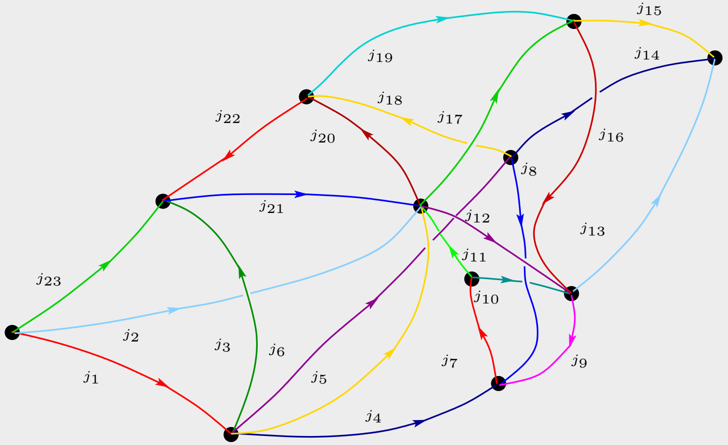
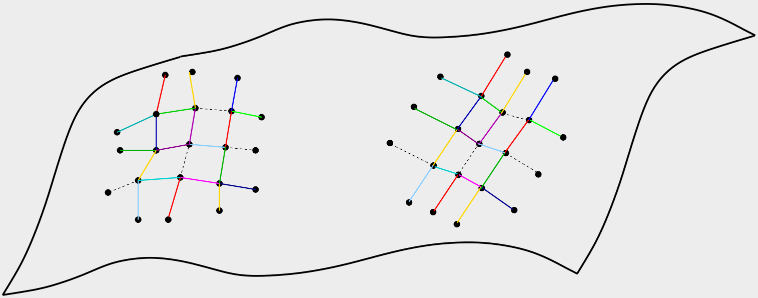
Basis of $\mathcal{H}_{kin}$

Spin network functions [Ashtekar, Isham, Lewandowski, Rovelli, Smolin '90]



LQG involves all embedded graphs

Embedded Graphs



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Summary Part I

Summary

- LQG tries to combine background independence & canonical quantisation rigorously
- Starts with Hamiltonian formulation of General Relativity
  - ADM variables are not suitable for quantisation
  - Introduction of Ashtekar variables becomes necessary
  - GR formulated in terms of holonomies and fluxes
- GR contains Gauß – , diffeomorphismen – and Hamiltonian – Constraint
- Dirac Quantisation: Constraints are implemented within the quantum theory
- Kinematical Hilbert space $\mathcal{H}_{\mathrm{kin}}$ as an intermediate step to $\mathcal{H}_{\mathrm{phys}}$
- Quantum – Einstein – Equations