Loop Quantum Gravity – Part II: Introduction

Kristina Giesel

NORDITA

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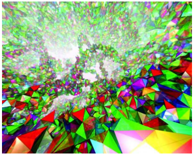

Plan of the Talk

- Part I:
  - Motivation for a theory of Quantum Gravity
  - Candidate: Loop Quantum Gravity
  - Classical starting point: General Relativity
  - General Relativity & background independence
  - Canonical quantisation of GR
  - Quantum – Einstein – Equations and their interpretation
- Part II:
  - Current research project within Loop Quantum Gravity
  - Geometrical operators (volume, area & length)
  - Solutions to Quantum – Einstein – Equations
- Summary & Conclusions

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Geometrical Operators: Volume, Length, Area

Volume Operator [Rovelli, Smolin '92 & Ashtekar, Lewandowski '93]

- Classical Volume $\mathrm{V_R} = \mathrm{V_R(E)}$
- Volume Operator $\widehat{\mathrm{V}}_\mathrm{R} = \mathrm{V_R}(\widehat{\mathrm{E}})$
- $\widehat{\mathrm{V}}_\mathrm{R}$ acts on $\psi(A)$
- Discrete spectrum
- Important also for $\widehat{\mathrm{H}}$
- Up to now purely kinematical result
  Could survive also dynamically

Volume of different regions

- $\mathrm{R_1} = 0$ $\mathrm{R_2} = 0$ $\mathrm{R_3} \neq 0$ $\mathrm{R_4} = 0$

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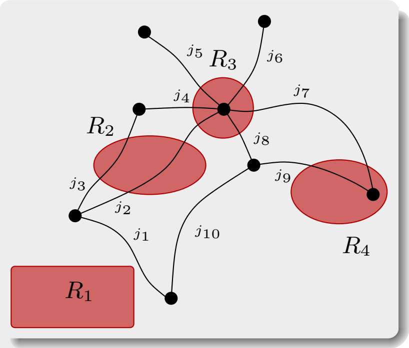

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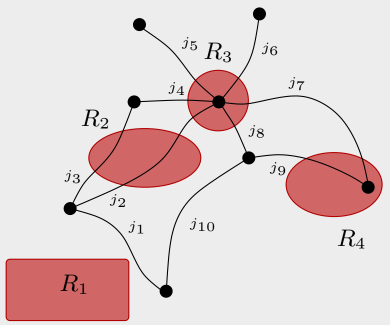

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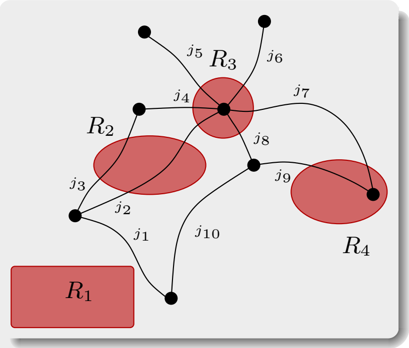

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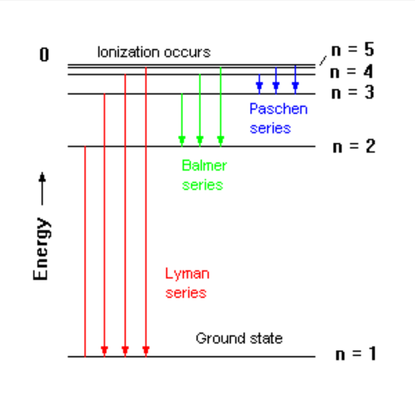
Hydrogen atom: Energy levels

Discrete Energy levels



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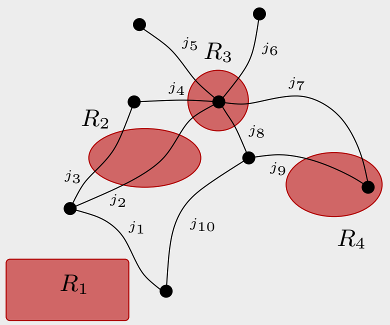

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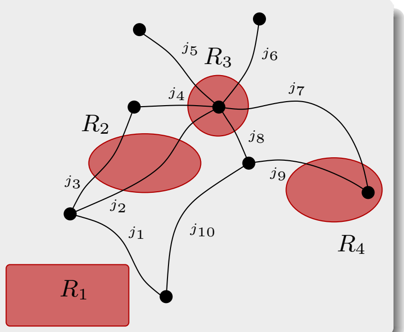

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Solutions of Quantum – Einstein – Equations

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- $\widehat{\mathrm{G}}\psi = 0, \quad \widehat{\mathrm{D}}\psi = 0$ can be solved, solutions live in $\mathcal{H}^{\mathcal{G}}_{\mathrm{Diff}}$
- More complicated: $\widehat{\mathrm{H}}\psi = 0$ yields to $\mathcal{H}_{\mathrm{phys}}$
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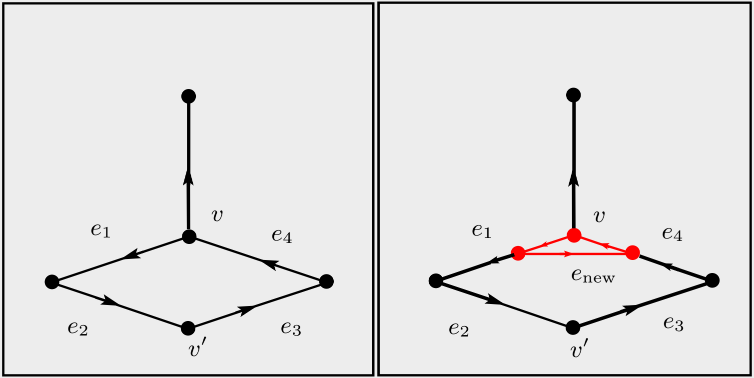
Graph – changing Operator

Graph – changing Hamiltonian Constraint Operator $\widehat{\mathrm{H}}$



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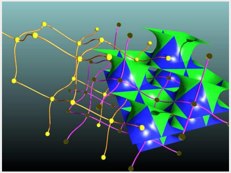

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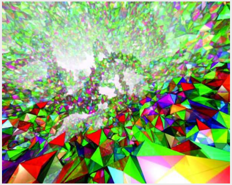

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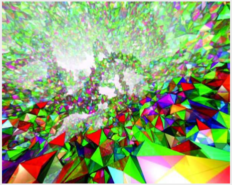

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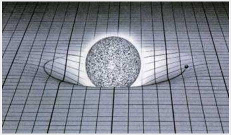

Loop Quantum Cosmology (LQC)

LQC: Symmetry Reduced Model of LQG [Kastrup, Bojowald '98]

- Assumes homogeneous and isotropic symmetry of cosmological FRW – models $\mathrm{ds}^2_{\mathrm{FRW}} = -\mathrm{dt}^2 + \mathrm{a(t)}(\mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2)$
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Knowledge about Quantum – Einstein – Equations

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Semiclassical Properties of Quantum – Einstein – Equations

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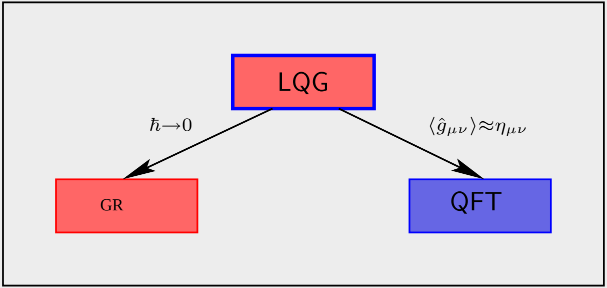
Consistency Check for any new Theory

Minimal Requirements for LQG



Hamiltonian constraint versus true Hamiltonian

- GR is a fully constrained theory $\mathrm{H_{can}} = \int_\sigma \mathrm{N(x)H(x) + N^a(x)D_a(x)} \approx 0$
- QFT with Standard Model has non – vanishing Hamiltonian $\mathrm{H_{SM}} \neq 0$
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- QFT with Standard Model has non – vanishing Hamiltonian $\mathrm{H}_{\mathrm{SM}} \neq 0$
- Conceptual problem
- Can we map GR into a true Hamiltonian system?

Reduced Phase Space Approach

Reduced instead of Dirac Approach

- Gauge dof are reduced at the classical level, Construction of observables
- Observables O in GR are gauge invariant quantities, that is $\{O, G_j\} = \{O, D_a\} = \{O, H\} \approx 0$
- Therefore also $\{O, H_{can}\} \approx 0 \longrightarrow$ problem of time in GR
- We need to find generator of their dynamics, physical Hamiltonian $\mathbf{H}_{phys} \neq 0$
- Only physical dof are quantised, algebra of observables, direct access to $\mathcal{H}_{phys}$

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Tasks to Do:

Reduced Phase Space Quantisation for LQG: Option 2

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- Task 2: Find the generator $\mathbf{H}_{\text{phys}}$ of their evolution
- Task 3: Quantisation of the classical reduced system

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Clocks for General Relativity

Explicit Construction of Observables for GR

- Problem of time in GR: Gauge and physical evolution, $\mathrm{H}_{\mathrm{can}} \approx 0$
- Physical evolution can be defined in relational way [Bergmann'50][Rovelli '90]
- Introduction of reference fields
- Choose clock and ruler to give time & space physical meaning
- Choose clocks which lead to (partially) deparametrised form of GR
- 4 scalar fields, 4 dust fields ... dynamically coupled observer

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Brown-Kuchař-Mechanism ['95]

Add Dust Lagrangian to Gravity + Standard Model

Dust action

$$S_{\rm dust} = -\frac{1}{2}\int_M d^4X\sqrt{|\det(g)|}\rho(g^{\mu\nu}U_\mu U_\nu + 1)$$

where $U_\mu = -T_{,\mu} + W_J S^J_{,\mu}$ is the four velocity, $J = 1, 2, 3$

- After solving second class constraints for ρ and W_J we are left with T, S^J
- $U^\mu = g^{\mu\nu}U_\nu$ is a geodesic, fields S^J are constant along geodesics, T defines proper time along each geodesic
- T becomes clock with values τ and S^j becomes ruler with values s^J
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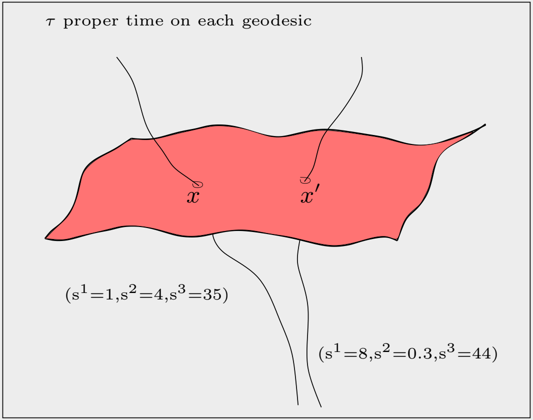
Observables with respect to Dust Clock & Rulers

Space time points are labelled by τ and s^j



Observables

Task 1: Observables: Gravity + any other standard matter

- Relational formalism [Rovelli '90],
  Power series expression for observables [Dittrich '05]
- For any f not depending on dust dof we construct observables

$$f \mapsto O_{f,\{T,S^J\}}(\tau, s^J), \quad q_{ab} \mapsto O_{A,\{T,S^J\}}(\tau, s^J)$$

- For simplicity use shorter notation:
  $O_{A,\{T,S^J\}}(\tau, s^J) := \mathbf{A}(\tau, s^J), \quad O_{E,\{T,S^J\}}(\tau, s^J) := \mathbf{E}(\tau, s^J)$

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Dynamics of Observables

Task 2: Physical Hamiltonian for GR

$$\mathbf{H}_{\rm phys} = \int_{\mathcal{S}} \mathrm{d}^3\mathrm{s}\,\mathbf{H}(\mathrm{s}) \quad \text{with} \quad \mathbf{H}(\mathrm{s}) = \sqrt{(\mathbf{H}^{\rm nd})^2 - \mathbf{Q}^{\mathbf{IJ}}\mathbf{D}^{\rm nd}_{\rm I}\mathbf{D}^{\rm nd}_{\rm J}}$$

$$\mathbf{H}_{\rm phys} \neq 0 \quad \text{is} \quad \text{non}-\text{vanishing}$$

$$\frac{\mathbf{F}(\tau,\mathrm{s})}{\mathrm{d}\tau} = \{\mathbf{H}_{\rm phys}, \mathbf{F}(\tau,\mathrm{s})\} \quad \text{true physical evolution}$$

$$\mathbf{H}_{\rm phys} \sim \int_{\mathcal{S}} \mathrm{d}^3\mathrm{s}\mathbf{H}^{\rm nd} + \dots = \mathrm{H}^{\rm SM} + \dots$$

Reduced Phase Space Quantisation

Task 3: Quantisation

- We need to find representation for observable holonomy – flux algebra $\{\mathbf{A}(\mathrm{e}), \mathbf{E}(\mathrm{S})\}$
- Additionally $\widehat{\mathbf{H}}_{\mathrm{phys}}$ must implementable as a well defined operator
- Possible to quantise the system with these requirements
- $\mathcal{H}_{\mathrm{phys}}$ looks very much like standard $\mathcal{H}_{\mathrm{kin}}$ from Dirac quantisation
- Quantum Einstein Equations

$$\frac{\mathrm{d}\hat{\mathbf{A}}(\mathrm{e})}{\mathrm{d}\tau} = [\hat{\mathbf{A}}(e), \widehat{\mathbf{H}}_{phys}], \quad \frac{d\hat{\mathbf{E}}(S)}{d\tau} = [\hat{\mathbf{E}}(S), \widehat{\mathbf{H}}_{phys}]$$

- Framework is based on combinatorial graphs, close to formulation of standard lattice gauge theory
- However, classical GR ist defined on a given manifold and thus needs embedding
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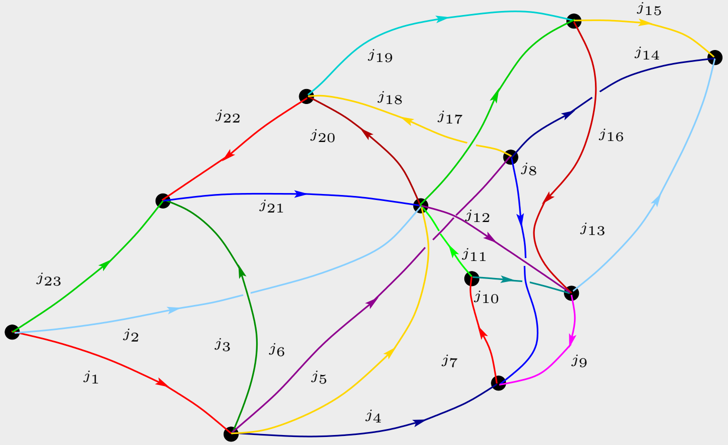
Basis of $\mathcal{H}_{kin}$

Spin network functions [Ashtekar, Isham, Lewandowski, Rovelli, Smolin '90]



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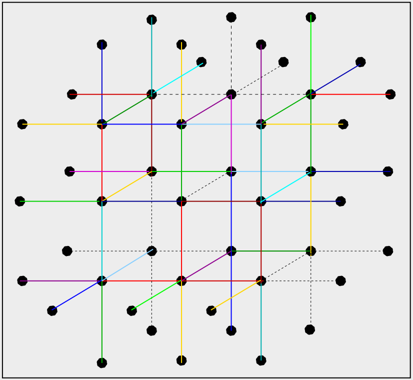
One infinite algebraic Graph

Algebraic graph with cubic topology



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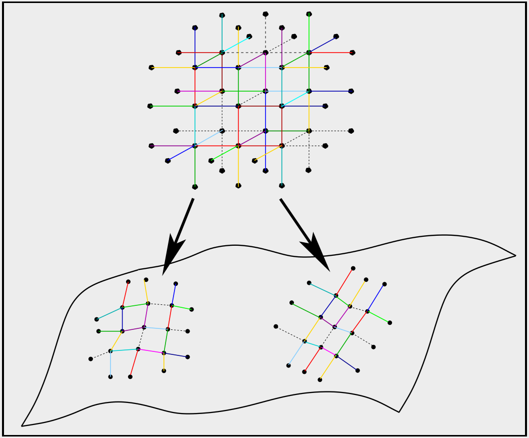
Fundamental Algebraic Graph

Information about embedding are encoded in coherent states



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Semiclassical Analysis of the Quantum – Einstein – Equations

Coherent States for LQG [Sahlmann, Thiemann, Winkler '03, Bahr '06]

- Only non – perturbative Techniques are allowed
- Coherent states for harmonic oscillator
- $z_0 := q_0 + ip_0, \hat{a} = \hat{q} + i\hat{p}$ then $\psi_{z_0} := e^{-\frac{|z_0|^2}{2}} \sum_{n=0}^{\infty} \frac{z_0^n}{\sqrt{n!}} |n\rangle$
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Semiclassical Analysis of the Quantum – Einstein – Equations

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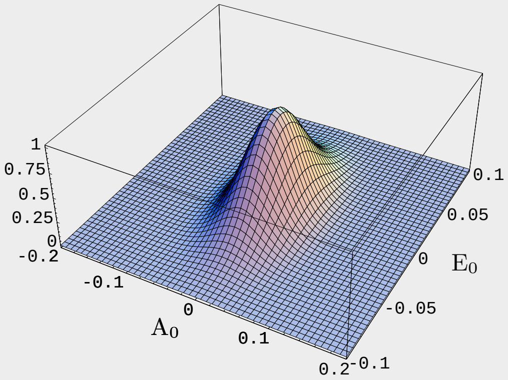
Coherent States

Coherent States around classical phase space point (A_0, E_0)



Semiclassical Limit [K.G., Thiemann 06 – '07]

Theorem: For any sufficiently fine $X(\alpha)$ and any (A_0, E_0)

1. Exp. value:

$$\langle \psi_{(A_0,E_0)} \, , \, \widehat{\mathbf{H}}_{\rm phys} \, \psi_{(A_0,E_0)} \rangle = \mathbf{H}_{\rm phys}(A_0, E_0) + O(\hbar)$$

2. Fluctuations: $\langle \widehat{\mathbf{H}}^2_{\rm phys} \rangle_{\psi_{(A_0,E_0)}} - \big(\langle \widehat{\mathbf{H}}_{\rm phys} \rangle_{\psi_{(A_0,E_0)}}\big)^2 = O(\hbar)$

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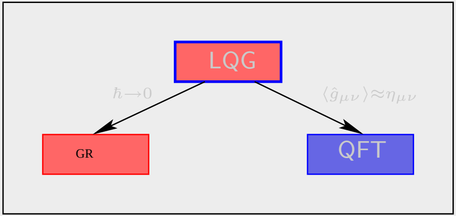

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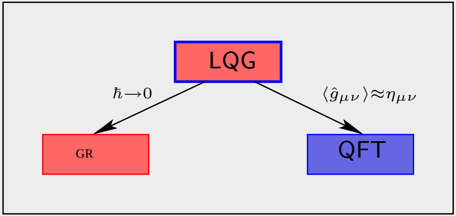

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  - By means of reference dust fields gauge dof can be reduced classically, gravitational Higgs
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- What happens in full LQG? Mimic quantum FRW spacetime
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