Non-thermal universe
Part II

From Fermi to DSA

Luke Drury
Dublin Institute for Advanced Studies

- Will discuss astrophysical acceleration mechanisms - how do cosmic accelerators work? - concentrating mainly on the class of Fermi processes.
- Motivation comes historically from cosmic ray observations going back to 1912 (and even a bit earlier) indicating the existence of an extremely energetic radiation of extraterrestrial origin.

"When, in 1912, I was able to demonstrate by means of a series of balloon ascents, that the ionization in a hermetically sealed vessel was reduced with increasing height from the earth (reduction in the effect of radioactive substances in the earth), but that it noticeably increased from 1,000 m onwards, and at 5 km height reached several times the observed value at earth level, I concluded that this ionization might be attributed to the penetration of the earth's atmosphere from outer space by hitherto unknown radiation of exceptionally high penetrating capacity, which was still able to ionize the air at the earth's surface noticeably. Already at that time I sought to clarify the origin of this radiation, for which purpose I undertook a balloon ascent at the time of a nearly complete solar eclipse on the 12th April 1912, and took measurements at heights of two to three kilometres. As I was able to observe no reduction in ionization during the eclipse I decided that, essentially, the sun could not be the source of cosmic rays, at least as far as undeflected rays were concerned."

From Victor Hess's nobel prize acceptance speech, December 12, 1936

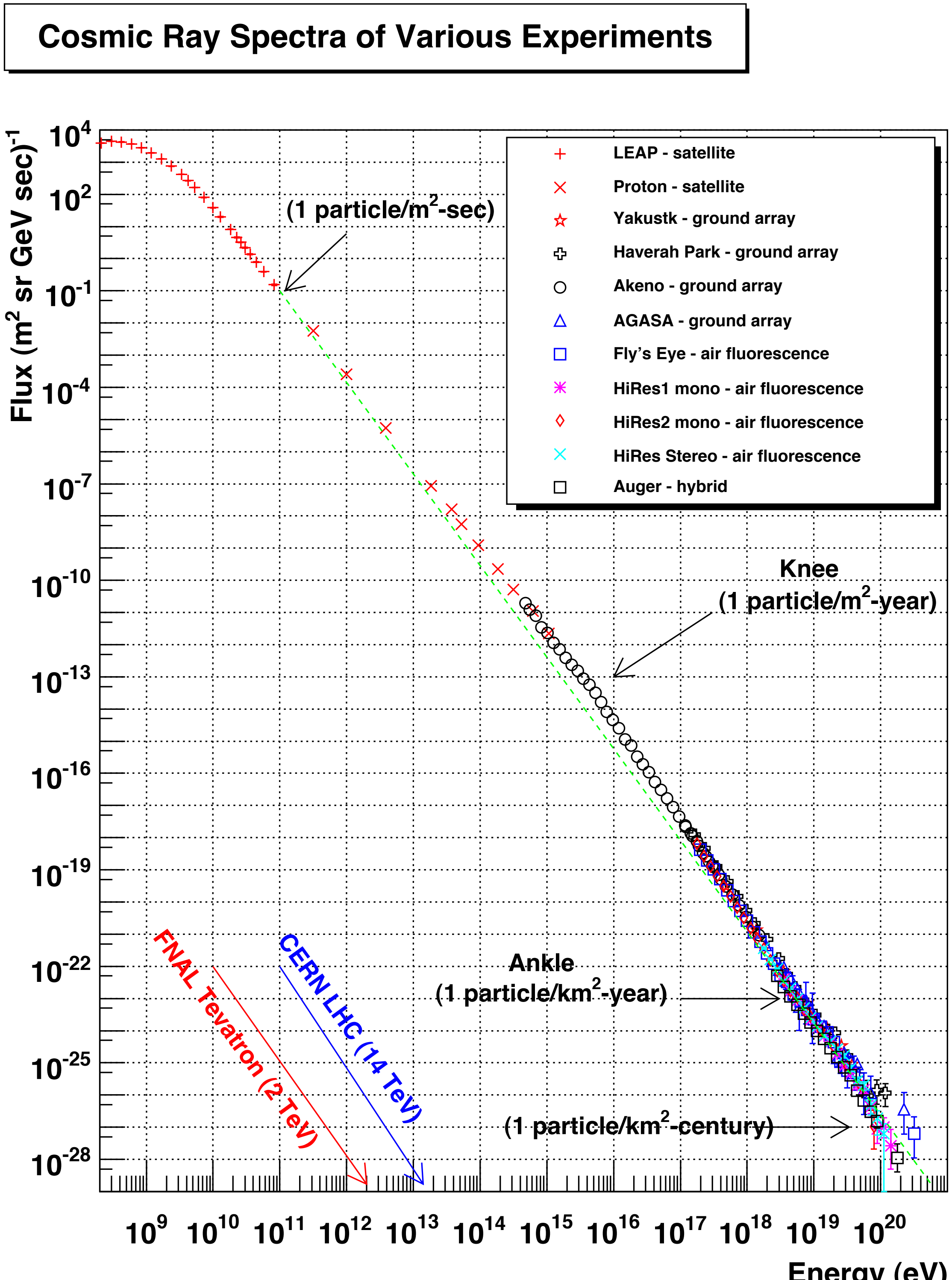


Figure courtesy of W Hanlon

Extraordinary energy range - from below a GeV to almost ZeV energies - and a remarkably smooth spectrum with only minor features at the "knee" and "ankle" regions. Almost perfect power-law over ten decades in energy and 30 decades in flux!

Beats CERN hollow even in centre of mass frame!

How and where does Nature do it?

In summary, need

- A very efficient Galactic accelerator
- Producing a hard power law spectrum over many decades
- Accelerating material of rather normal composition
- Not requiring very exotic conditions

Astrophysical Accelerators

- Major problem - most of the universe is filled with conducting plasma and satisfies the ideal MHD condition $E + U \wedge B = 0$
- Locally no E field, only B
- B fields do no work, thus no acceleration!

Two solutions

- Look for sites where ideal MHD is broken (magnetic reconnection, pulsar or BH environment, etc)
- Recognise that E only vanishes locally, not globally, if system has differential motion - this is the class of Fermi mechanisms on which I will concentrate.

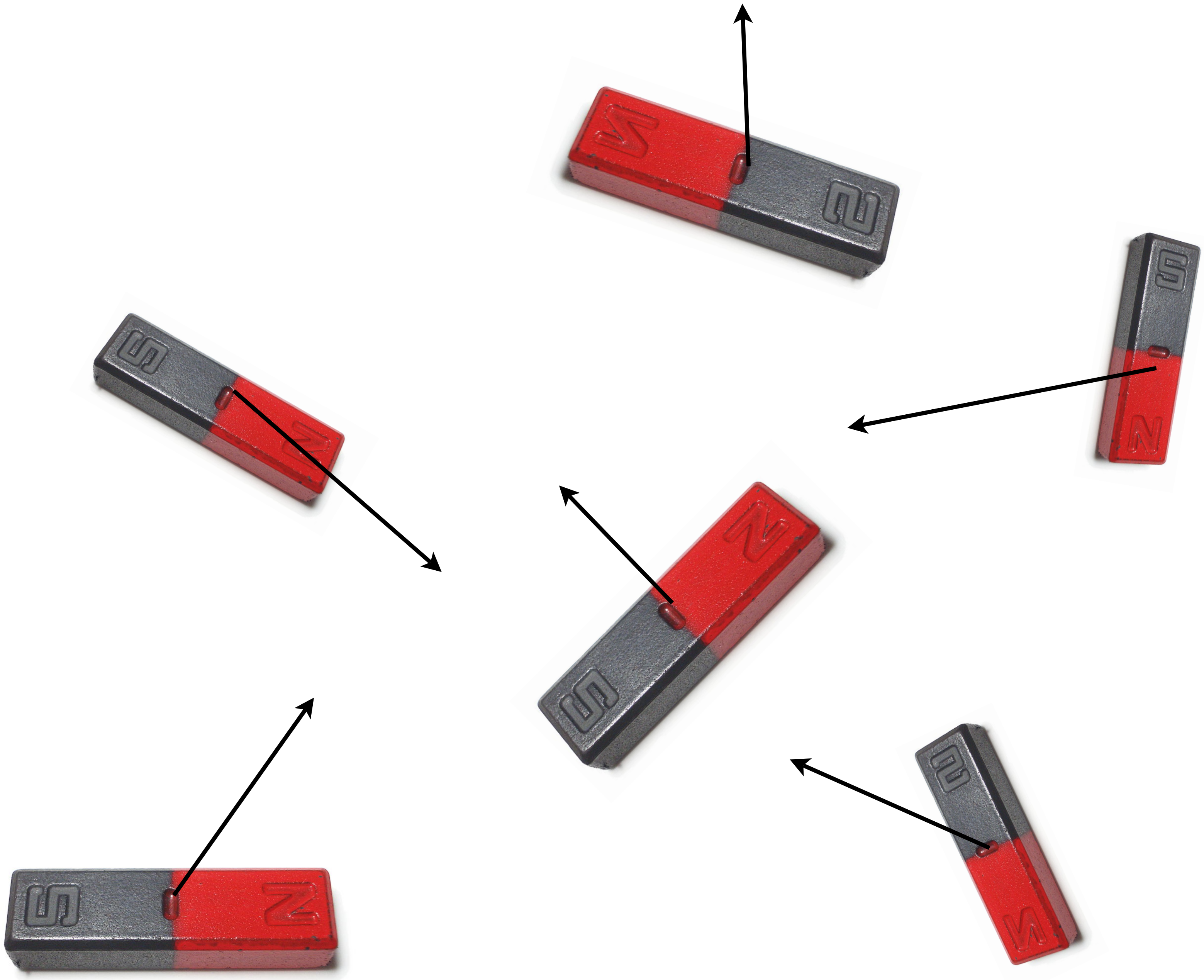
- Close analogy to terrestrial distinction between
  - One shot electrostatic accelerators
  - Storage rings with many small boosts

Fermi 1949

- Galaxy is filled with randomly moving clouds of gas.
- The clouds have embedded magnetic fields.
- High-energy charged particles can “scatter” off these magnetised clouds.
- The system will attempt to achieve “energy equipartition” between macroscopic clouds and individual atomic nuclei leading to acceleration of the particles.

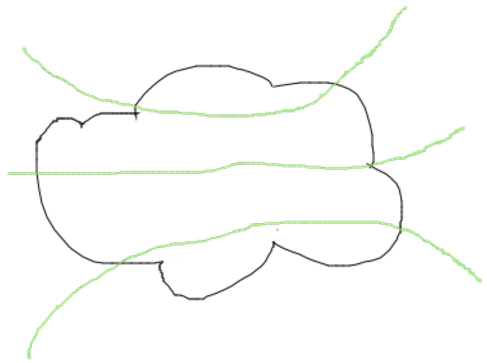
Fermi's insight

- Magnetic fields couple microscopic degrees of freedom of individual charged particles to macroscopic bulk motion of plasma.
- Attempt to achieve equilibrium inevitably leads, on average, to energy transfer to particles.
- *Gedanken* experiment - think of "gas" of bar magnets plus a few protons....

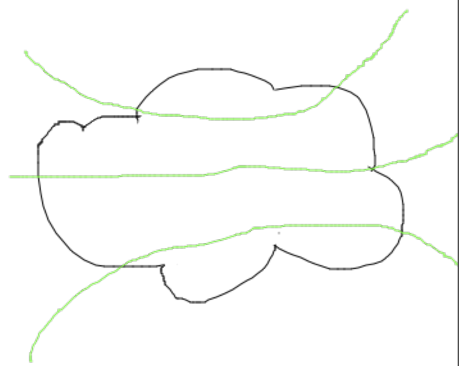

S
N

Head-on collision resulting in energy gain

Head-on collision resulting in energy gain



But of course also tail-on energy loss collisions...



But of course also tail-on energy loss collisions...

Trivial but very important point; the energy of a particle is not a scalar quantity, but the time-like component of its energy-momentum four vector. If we shift to a different reference frame, the energy changes and so does the magnitude of the momentum.

Shift from lab frame to frame of cloud moving with velocity $\vec{U}$

$$E' = \frac{E + \vec{p} \cdot \vec{U}}{\sqrt{1 - U^2/c^2}}$$

$$\begin{aligned}
\Delta E &\approx \vec{p}\cdot\vec{U} \\
\Delta p &\approx \frac{E}{c^2 p}\vec{p}\cdot\vec{U} = \frac{1}{v}\vec{p}\cdot\vec{U} \\
&\approx \frac{1}{c}\vec{p}\cdot\vec{U} \qquad p \gg mc \\
&\approx \frac{m}{p}\vec{p}\cdot\vec{U} \qquad p \ll mc
\end{aligned}$$

Lab frame → Cloud frame → Lab frame

$$\Delta p \approx \beta p\,(\cos\vartheta_1 - \cos\vartheta_2)$$

for relativistic particles scattering off clouds with dimensionless peculiar velocity $\beta = \frac{U}{c}$

Mean square change in momentum is

$$\langle \Delta p^2 \rangle = \frac{2}{3}\beta^2 p^2$$

Particle makes a random walk in momentum space with steps of order βp at each scattering.

Corresponds to diffusion process,

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right) + Q - \frac{f}{T}$$

with diffusion coefficient of order

$$D_{pp} \approx \frac{\beta^2 p^2}{\tau}$$

where τ is the mean time between scatterings.

Fermi pointed out that if the scattering and loss time scales are both energy independent this produces power law spectra with exponent

$$q = \sqrt{\frac{\tau}{\beta^2 T} + \frac{9}{4}} - \frac{3}{2}$$

Beautiful but wrong

- Too slow - acceleration time scale is diffusion time scale

$$\frac{p^2}{D_{pp}} \approx \frac{\tau}{\beta^2}$$

$$\beta \leq 10^{-4}, \qquad \tau \geq 1\,\mathrm{yr}$$

$$\geq 10^8\,\mathrm{yr}$$

- Requires unnatural fine-tuning of collision time and loss time to produce a power-law.
- Requires an additional injection process to get particles to relativistic energies (very high energy loss rate for non-relativistic charged particles).
- Would imply that higher energy particles are older, contrary to the observed secondary to primary ratios.
- Has difficulty with the chemical composition.

But...

- Must occur at some level (*cf* reacceleration models of CR propagation)
- Is historically important
- Contains valuable physical insights

General cosmic ray transport equation

$$\frac{\partial f}{\partial t} + U \cdot \nabla f =$$ Convective derivative

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right)$$ Momentum diffusion

$$+ \nabla \cdot (D_{xx} \nabla f)$$ Spatial diffusion

$$- \frac{1}{3}\nabla \cdot U\, p\frac{\partial f}{\partial p}$$ Adiabatic compression

$$+ Q - \frac{f}{T}$$ Sources and sinks

Key Assumptions

- Distribution function is close to isotropic (strong scattering by magnetic fields)

$$f(\vec{p}) \approx f(p), \qquad p = |\vec{p}|$$

- Mixed coordinate system, particle momentum p measured in local fluid frame, fluid velocity U in global reference system.

- Motion is non-relativistic $U \ll c$

If the same scattering gives rise to both the momentum and spatial diffusion, the two coefficients are related roughly by

$$D_{pp} \approx \frac{\beta^2 p^2}{\tau}$$

$$D_{xx} \approx \frac{\lambda^2}{\tau} = c^2 \tau$$

$$D_{pp} D_{xx} \approx V^2 p^2$$

where V is the random velocity of the scattering centres, often taken to be Alfvén waves. Thus if one is large, the other is small and *vice versa.*

Shock acceleration

- Major breakthrough in 1977/1978
- Four independent publications of same essential idea by
  - Krymsky
  - Blandford and Ostriker
  - Axford, Leer and Skadron
  - Bell

Collisionless Shocks

- Shocks, sudden jumps in velocity and density, appear whenever flow hits an obstacle, flows collide, flows converge.
- Physically appear as 1-D dissipative structures in which KE of bulk motion is transferred to micro-scale random motion.
- Dissipation in collisionless shock comes from collective plasma processes (not, as in gas dynamics, from 2-body collisions).

Keep advection, adiabatic compression and spatial diffusion terms in transport equation,

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \nabla f = \nabla(\kappa \nabla f) + \frac{1}{3}(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p}$$

and apply it to the flow through a shock

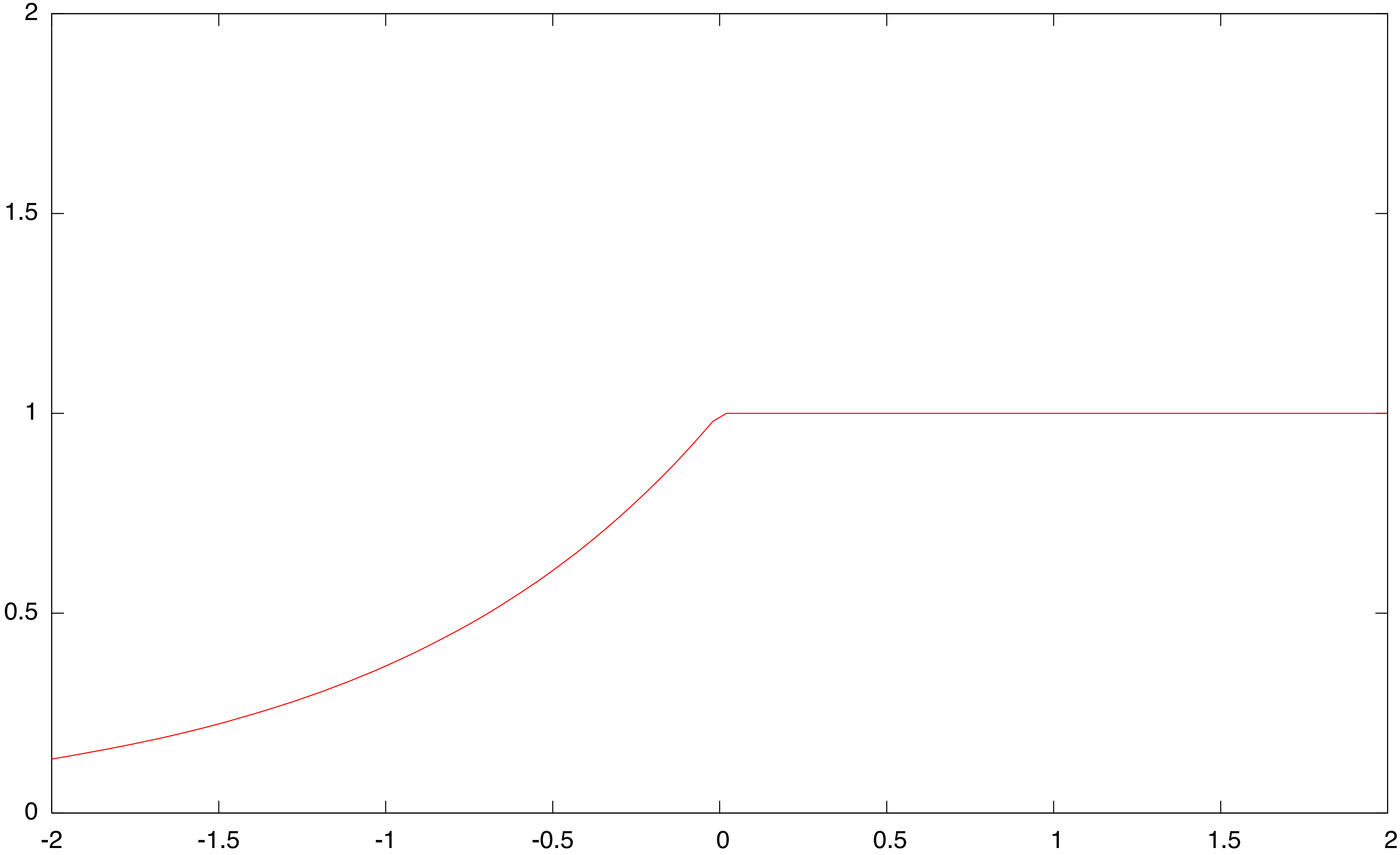
$$U(x) = \begin{cases} U_1, & x < 0 \\ U_2, & x > 0 \end{cases}$$

Look for steady solutions in upstream and downstream regions...

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \nabla f = \nabla(\kappa \nabla f) + \frac{1}{3}(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p}$$

$$f(x,p) = f_0(p) \exp \int \frac{U}{\kappa} dx \qquad x \leq 0$$

$$f(x,p) = f_0(p) \qquad x \geq 0$$


2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \nabla f - \frac{1}{3}(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p} = \nabla(\kappa \nabla f)$$

Advection in x-space with spatial velocity $\vec{U}$

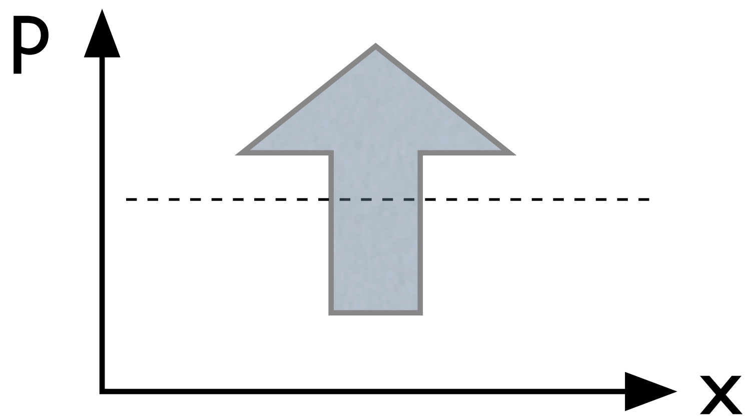
Advection in p-space with velocity given by $-\frac{1}{3}\left(\nabla \cdot \vec{U}\right) p$

Acceleration from compression in shock front!

Useful to think in terms of the acceleration flux,

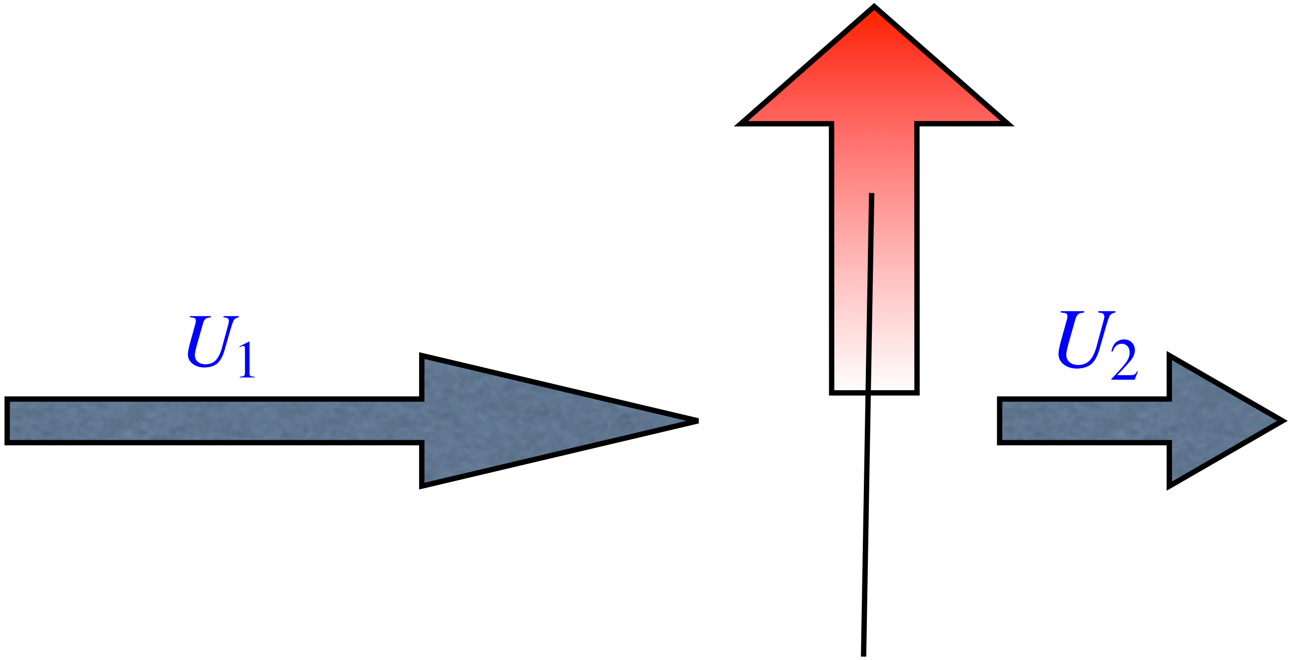
$$\Phi(p) = \int \frac{4\pi p^3}{3} f(p)(-\nabla \cdot \vec{U})\, d^3x$$

Rate at which particles are being accelerated through a given momentum (or energy) level.



If compression occurs only at the shock, then

$$\Phi(p) = \frac{4\pi p^3}{3} f_0(p)\,(U_1 - U_2)$$



and is localised at the shock.

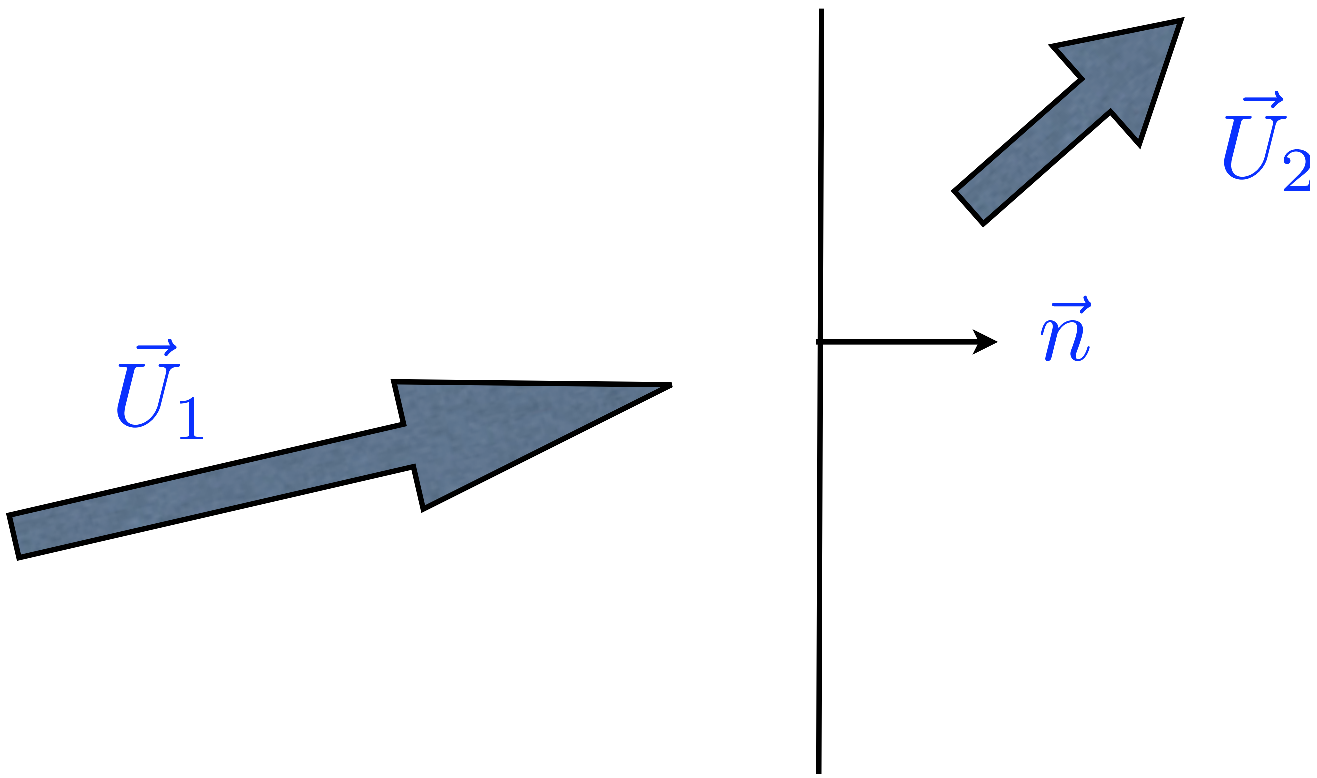
Formally follows from putting

$$-\nabla \cdot U = (U_1 - U_2)\delta(x)$$

in the transport equation, but can be seen more directly by looking at the kinetic level.

$$\begin{aligned}
\Phi(p) &= \int \frac{1}{v}\vec{p}\cdot\left(\vec{U}_1 - \vec{U}_2\right)(\vec{v}\cdot\vec{n})\, f(p)p^2\, d\Omega \\
&= p^3 f(p)\,\vec{n}\cdot\left(\vec{U}_1 - \vec{U}_2\right)\int_{-1}^{+1}\mu^2\, 2\pi d\mu \\
&= \frac{4\pi}{3}p^3 f(p)\,\vec{n}\cdot\left(\vec{U}_1 - \vec{U}_2\right)
\end{aligned}$$

This result applies quite generally to oblique MHD shocks and only depends on the near isotropy of the particle distribution at the shock and the condition (related to the isotropy) that the particles are fast relative to the flow.



- Positive, though small, change in momentum each time shock is crossed in either direction of order $\Delta U/v$

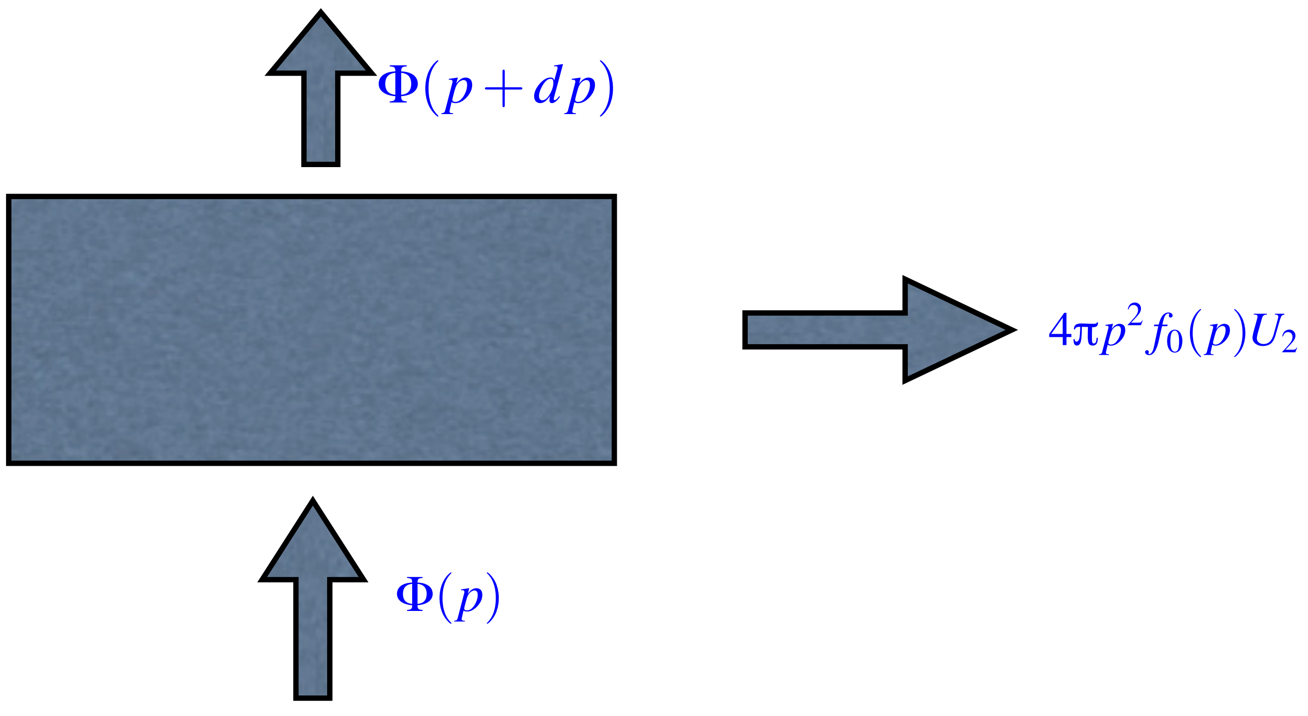
- In diffusion regime particles cross shock many times - probability of escape downstream is low, $v/4U_2$

Is it a con trick?

- Nothing happens to particle as it crosses the front - we just change the reference frame.
- But if we were to work in the shock frame, then the scattering processes would all be energy changing.
- Using separate reference frames up and down stream is consistent and greatly simplifies the analysis by concentrating all the effects at the shock.

Now write down particle conservation law for balance between rate of advection away from shock region and acceleration

$$\frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$



Particles interacting with the shock fill a “box” extending one diffusion length upstream and downstream of the shock,

$$L = \left(\frac{\kappa_1}{U_1} + \frac{\kappa_2}{U_2} \right)$$

so time dependent particle conservation is

$$\frac{\partial}{\partial t} \left(4\pi p^2 f_0(p) L \right) + \frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$

or, substituting for the acceleration flux

$$4\pi p^2 L \frac{\partial f}{\partial t} + 4\pi p^2 f(U_1 - U_2) + \frac{4\pi p^3}{3}(U_1 - U_2)\frac{\partial f}{\partial p} = -4\pi p^2 f U_2$$

and simplifying

$$L\frac{\partial f}{\partial t} + \frac{U_1 - U_2}{3} p \frac{\partial f}{\partial p} = -U_1 f$$

"Box" approximation to shock acceleration -
can be trivially solved by method of characteristics

The single PDE

$$L\frac{\partial f}{\partial t} + \frac{U_1 - U_2}{3} p \frac{\partial f}{\partial p} = -U_1 f$$

is equivalent to the pair of ODEs

$$\begin{aligned} \frac{d\,p}{d\,t} &= \frac{U_1 - U_2}{3L} p \\ \frac{d\,f}{d\,p} &= -3\frac{U_1}{U_1 - U_2}\frac{f}{p} \end{aligned}$$

The first equation says that particles gain energy at rate,

$$t_{\rm acc} = \frac{p}{\dot{p}} = \frac{3L}{U_1 - U_2}$$

the second that the number of particles decreases in such a way as to give a power-law spectrum as a function of momentum,

$$f \propto p^{-3U_1/(U_1-U_2)}$$

Main defect of the box model is that it assumes that all particles gain energy at precisely the same rate, whereas in reality there is considerable dispersion in the acceleration time distribution. However it is a useful simplification that captures much of the physics. Can add synchrotron losses, spherical geometry etc without too much difficulty.

It is actually possible to do a lot analytically with the full transport equation, and it is quite easy to solve numerically, so this linear test-particle theory is very well understood.

Key points

- Process is a pure first-order acceleration (although not if post-shock expansion is included).
- Naturally produces power-law spectra with exponent fixed by kinematics of shock (scale free).
- Spectral exponents are in right ball park.

- Process is relatively fast if local turbulence at shock is high (as is expected from plasma instabilities) and these scatter particles strongly - usual assumption is Bohm scaling.

$$\kappa \approx \frac{1}{3} r_g v$$

$$t_{\rm acc} \approx 10 \frac{\kappa_1}{U_1^2}$$

$$\dot{E} = v\dot{p} = \frac{vp}{t_{\rm acc}} \approx 0.3 e B U_1^2$$

For typical ISM field of 0.3nT and a young SN shock of velocity 3000 km/s get energy gain of 1000eV/s

$$0.3\mathrm{e} \times (3 \times 10^{-10}\,\mathrm{T}) \times (3 \times 10^{6}\,\mathrm{m\,s^{-1}})^2 = 10^3\,\mathrm{eV/s}$$

Impressive, and easily enough to overcome coulomb losses etc, but do not expect such high shock speeds to last more than a few hundred years. After 300 years maximum energy still only of order 10TeV, well short of the PeV needed for the knee region (Lagage and Cesarsky limit).

One of very general limits on possible accelerators:

$$r_g = \frac{p}{eB} < L \implies E < eBLc$$

If electric field derived from a velocity scale U operating over a length scale L:

$$E \leq eUBL$$

Diffusive shock acceleration in Bohm limit

$$E \leq 0.3eBU^2t = 0.3eBUL$$

Basically about as fast as is physically possible!

Usefully summarized in so-called Hillas plot of typical size of astronomical systems versus magnetic field strength.

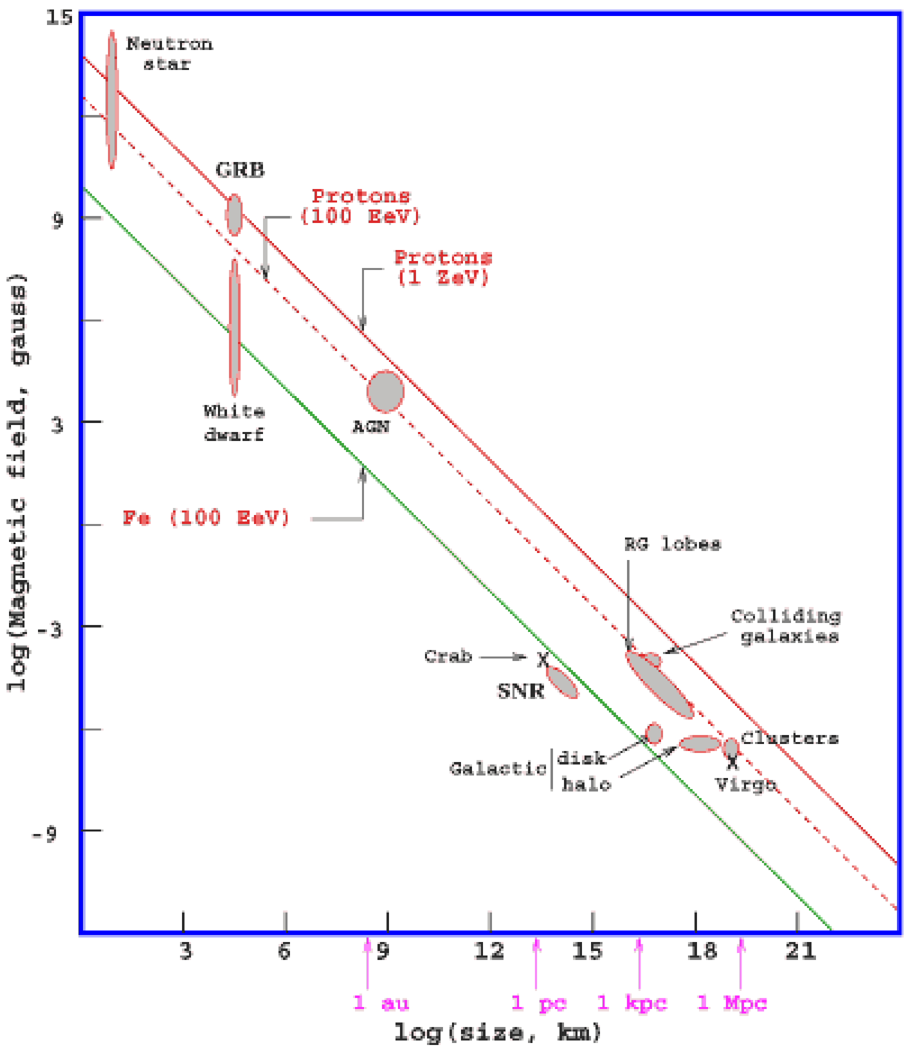
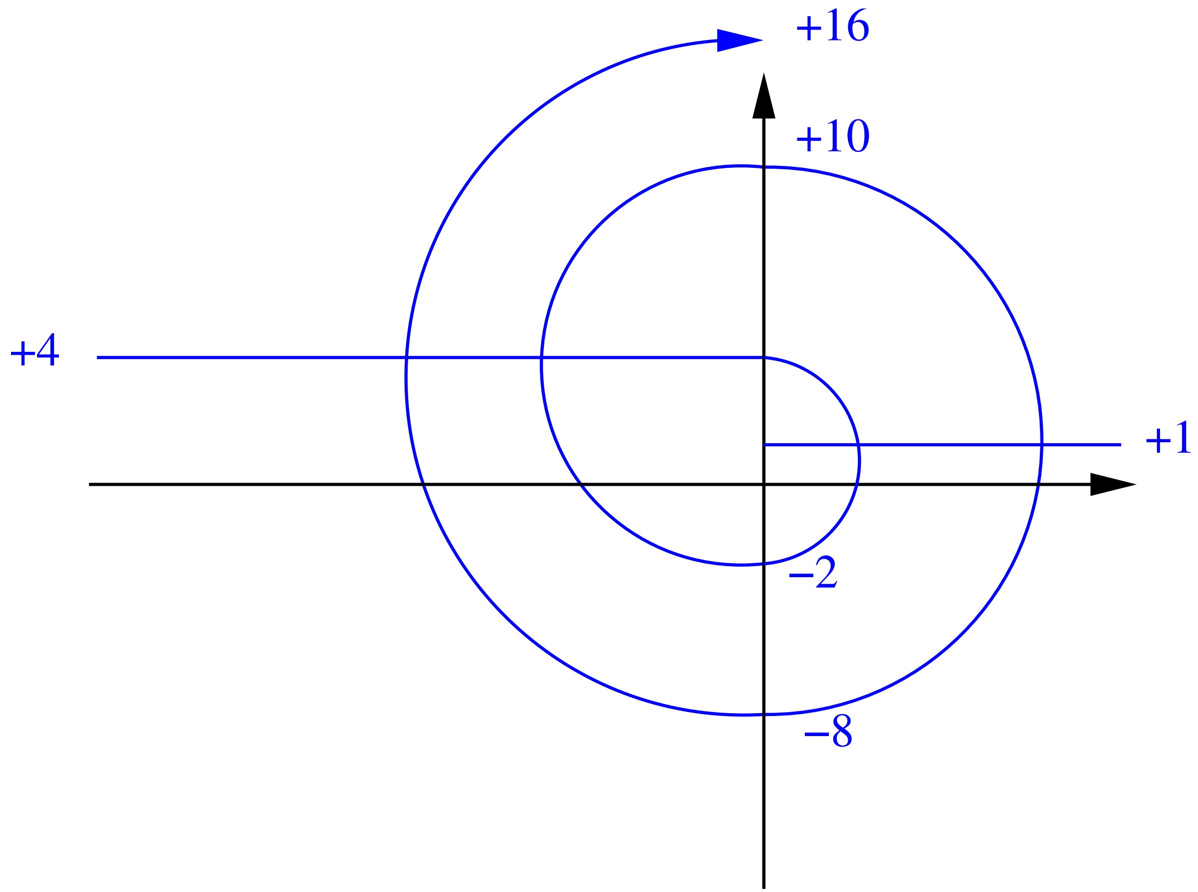


Figure 5. Size of astronomical objects versus their typical magnetic field (from A. H. Hillas [21]).

Injection

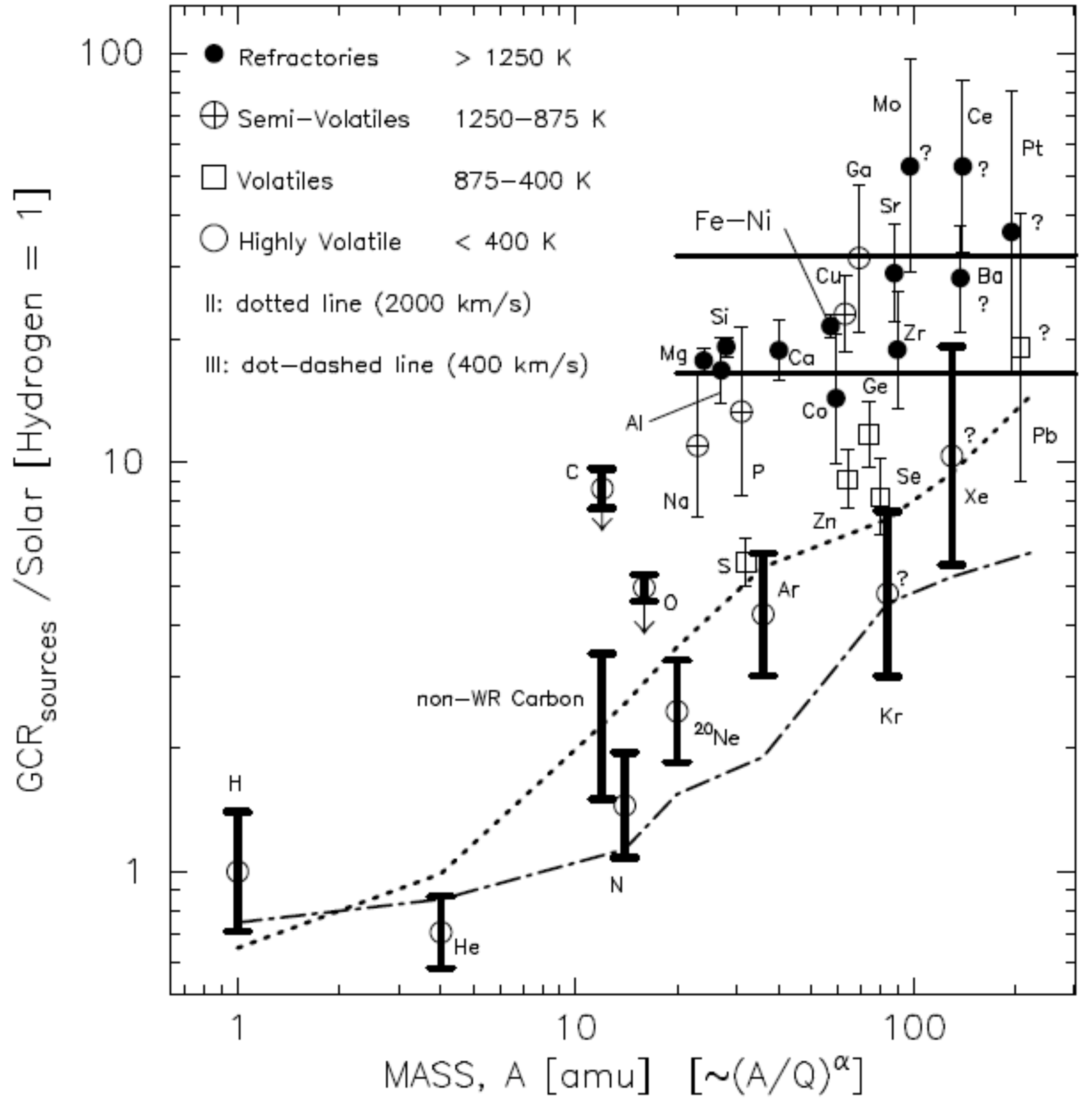
- Second great advantage of DSA is that it does not need a separate injection process - the shock can directly inject particles into the acceleration process.
- Although distributions are anisotropic at these low energies, same basic process of shock crossing and magnetic scattering should occur.



Have back-streaming ions for compression > 2.

- Well known in hybrid simulations of collisionless shocks (and more recently in PIC simulations also).
- Few backstreaming ions then act as seed population for further acceleration.
- NB electron injection is much more complicated, but there are certainly possible processes which can produce sufficiently energetic electrons.

- Expect injection to be easiest for high rigidity species - compositional bias towards heavy ions.
- Fits qualitatively with the observed CR composition, but hard to make it work quantitatively unless dust is included.
- With limited acceleration and sputtering of dust grains can get very good fit to observed composition.



From Ellison, Drury and Meyer (1997) ApJ 487 197

- Real problem is to throttle back the injection of ions - easy to see that for typical SNR shocks if more than about 0.0001 of incoming protons become relativistic cosmic rays there is a severe energy problem!

For shock at 3000 km/s, 1% speed of light, mean kinetic energy per incoming particle is

$$10^{-4} m_p c^2$$

mean energy per CR several times $m_p c^2$

Is Fermi:

- A large space mission
- A class of acceleration theories
- A unit of length
- A famous Italian scientist
- All of the above