The non-thermal universe 1

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Non-thermal?

- Generally, systems far from equilibrium
- Specifically, systems where a few particles have energies very much greater than the average ``thermal" energy.

$$E \gg kT$$

(Paradigmatic example - cosmic rays)

- Normal matter, unless ultradense, can not be heated much above the electron rest mass energy (adding extra energy just goes into pair production).
- Thus the universe above 1 MeV is effectively completely non-thermal.
- There are non-thermal effects at lower energies, but in non-thermal astroparticle physics we generally deal with situations where at least the electrons are relativistic.

- mostly also consider situations where the particles are charged and the long-range interactions between particles are electromagnetic (plasma astrophysics).

- Will not discuss neutrino physics, nuclear reactions, etc much in these lectures - these aspects will be covered by others.

- Will discuss basic plasma physics of non-thermal particles, acceleration etc.

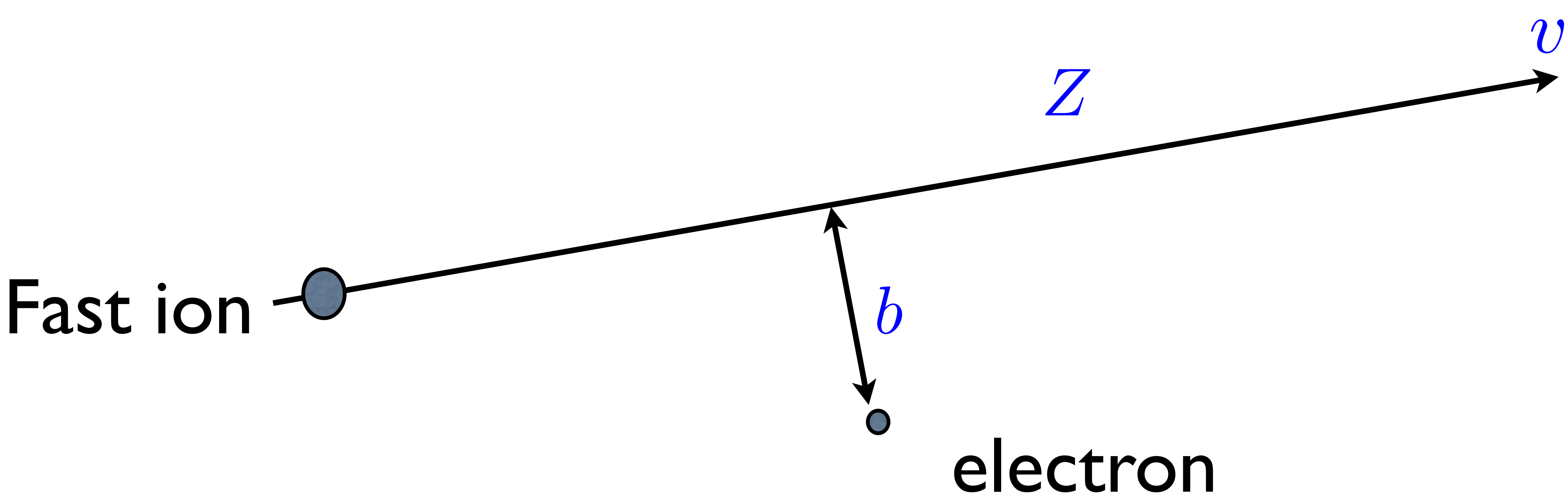
Recommended literature

- M. S. Longair, High energy astrophysics
- Rybiki and Lightman, Radiative processes in Astrophysics
- Spitzer, Physical processes in the interstellar medium

Time scales and energy loss processes

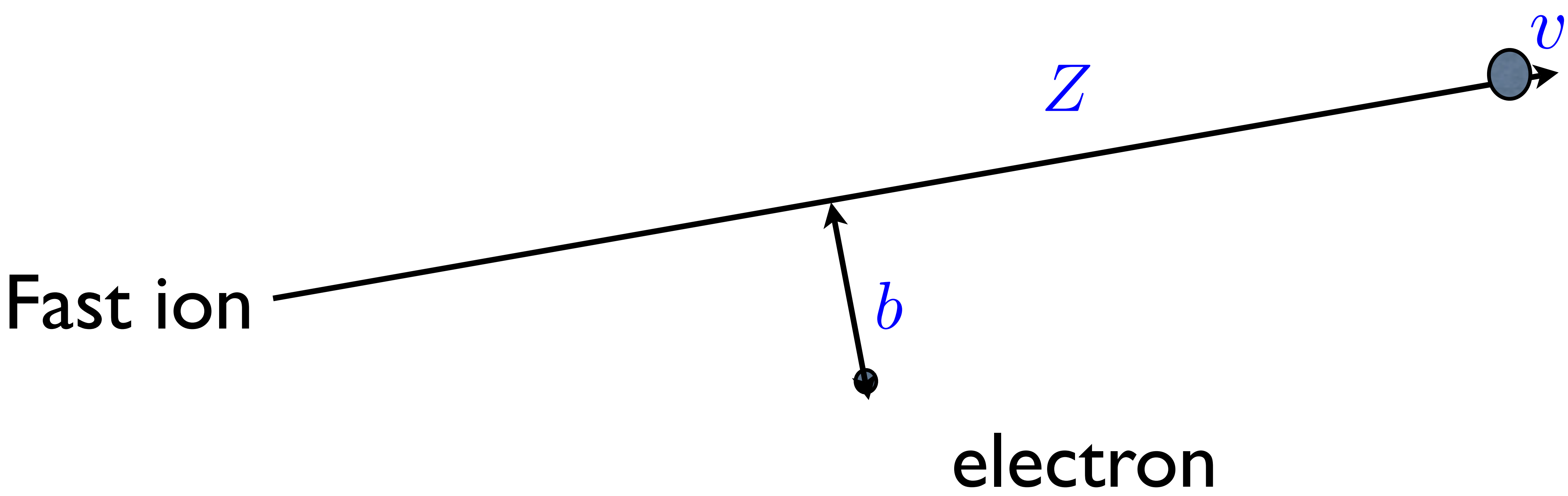
- Need to estimate energy transfer time scales from `fast' particles to background `thermal' particles.
- How long can a non-thermal particle survive under normal astronomical conditions?
- start with `ionization loss'.

Impulse approximation estimate.



Impact parameter	b
Ion velocity	v
Ion charge	Z

Impulse approximation estimate.



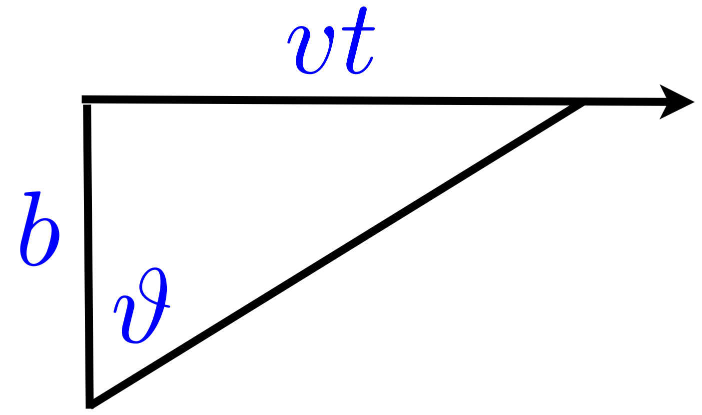
Impact parameter b

Ion velocity v

Ion charge Z

x-component of acceleration of the electron is

$$m_e \ddot{x} = \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{b^2 + v^2 t^2} \cos\vartheta$$



$$b^2 + v^2 t^2 = b^2 (1 + \tan^2 \vartheta) = \frac{b^2}{\cos^2 \vartheta}$$

and momentum transfer is thus

$$
\begin{aligned}
\Delta p = m_e \dot{x} &\approx \frac{Ze^2}{4\pi\epsilon_0} \int_{-\infty}^{+\infty} \frac{dt}{b^2 + v^2 t^2} \cos\vartheta \\
&= \frac{Ze^2}{4\pi\epsilon_0} \int_{-\pi}^{+\pi} \frac{\cos^3\vartheta}{b^2} d\left(\frac{b\tan\vartheta}{v}\right) \\
&= \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{bv} \int_{-\pi}^{+\pi} \cos\vartheta d\vartheta \\
&= \frac{Ze^2}{2\pi\epsilon_0} \frac{1}{bv}
\end{aligned}
$$

Energy transfer per collision is

$$\Delta E = \frac{(\Delta p)^2}{2m_e} = \left(\frac{Ze^2}{2\pi\epsilon_0}\right)^2 \frac{1}{2m_e} \left(\frac{1}{bv}\right)^2$$

Collision rate at impact parameter b is

$$v\, n_e\, 2\pi b\, db$$

and so

$$\frac{\Delta E}{\Delta t} = n_e \left(\frac{Ze^2}{2\pi\epsilon_0}\right)^2 \frac{1}{2m_e} \int \left(\frac{1}{bv}\right)^2 2\pi b v\, db$$

Logarithmic divergence at both ends!

$$\frac{\Delta E}{\Delta t} = \frac{n_e Z^2}{m_e v} \frac{e^4}{4\pi\epsilon_0^2} \int \frac{db}{b}$$

Coulomb logarithm

$$\lambda \equiv \ln \Lambda \equiv \int \frac{db}{b} = \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

for order of magnitude estimates can get away with very crude estimates of limits.

Two limits on minimum impact parameter...

Classical; impulse approximation breaks down if interaction is strong,

$$\frac{Ze^2}{4\pi\epsilon_0 b_{\min}} \approx T_{\rm CM} \approx \frac{1}{2} m_e v^2$$

Quantum; Heisenberg uncertainty in electron location

$$b_{\min} m_e v \geq \hbar$$

$$b_{\min} = \max\left(\frac{\hbar}{m_e v}, \frac{Ze^2}{2\pi\epsilon_0 m_e v^2}\right)$$

Normally QM limit is dominant...

$$\frac{(b_{\min})_Q}{(b_{\min})_C} = \frac{\hbar}{m_e v}\frac{2\pi\epsilon_0 m_e v^2}{Ze^2}$$

$$= \frac{1}{2\alpha}\frac{v}{cZ}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 c\hbar} \approx \frac{1}{137}$$

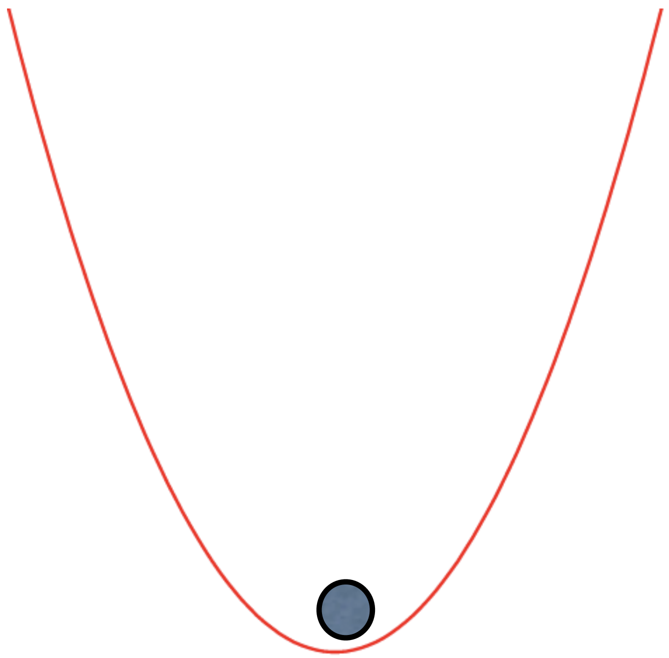
Also two forms of limits on maximum impact parameter:

For fast particles moving through neutral matter, can electrons be treated as free? Actually in bound state of atoms. Model as SHO with characteristic frequency ω_0.

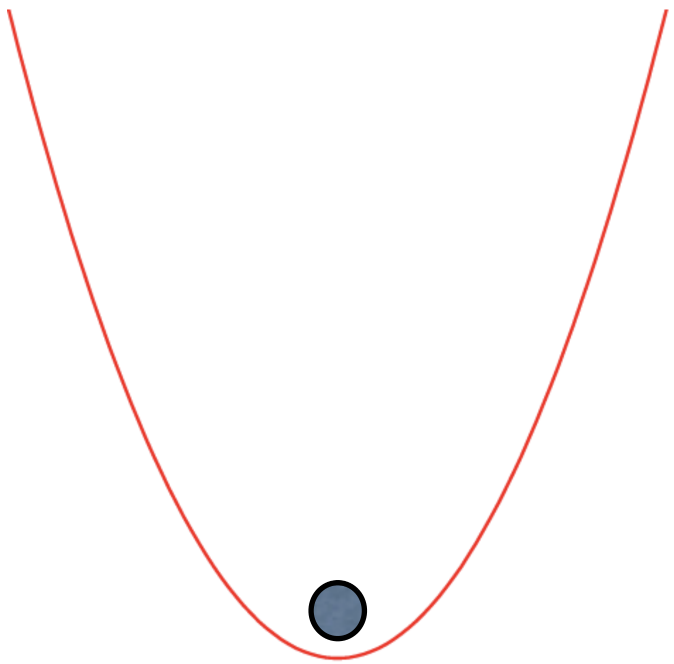
Field of passing ion is a slow adiabatic change if

$$\frac{2b}{v} \gg \frac{1}{\omega_0}$$

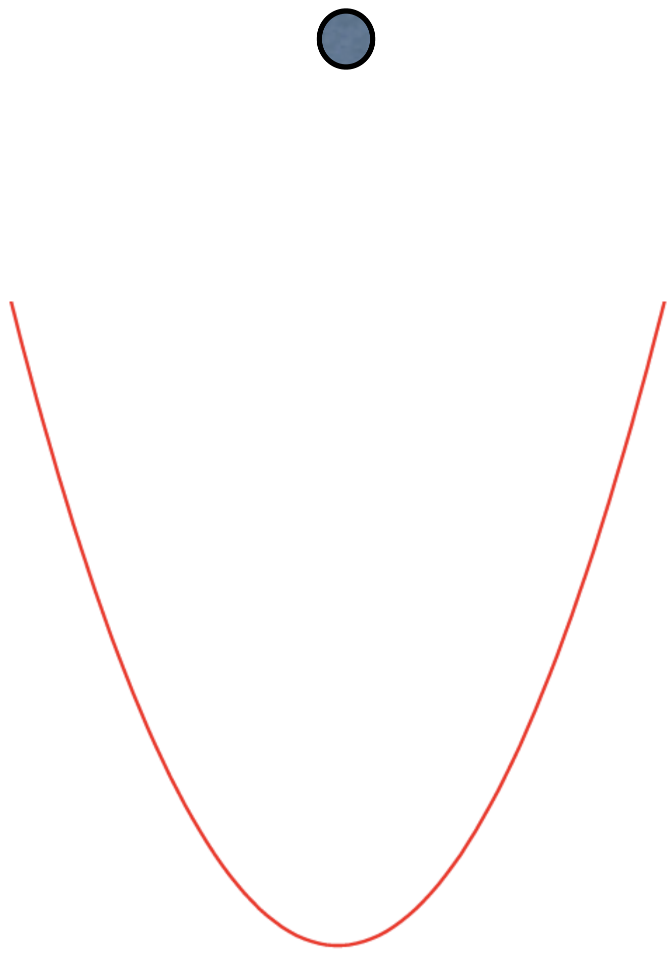
and little or no energy transfer



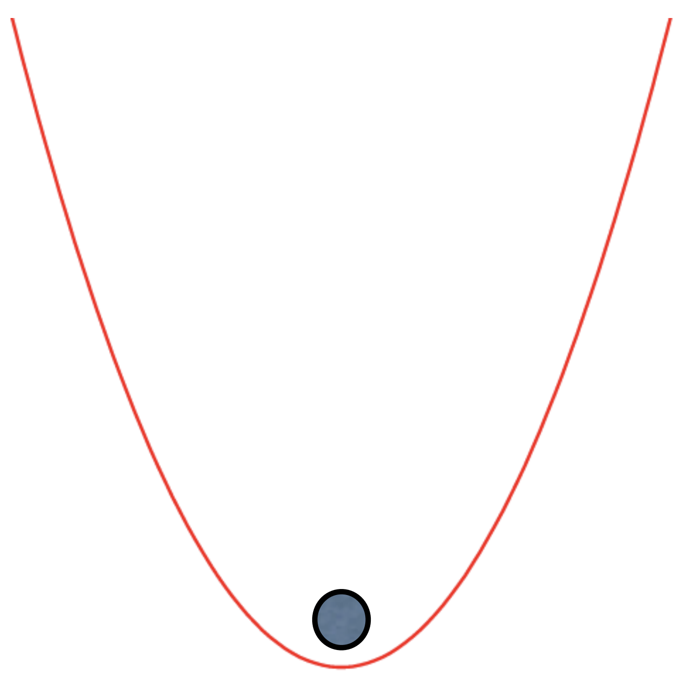
Fast - excitation



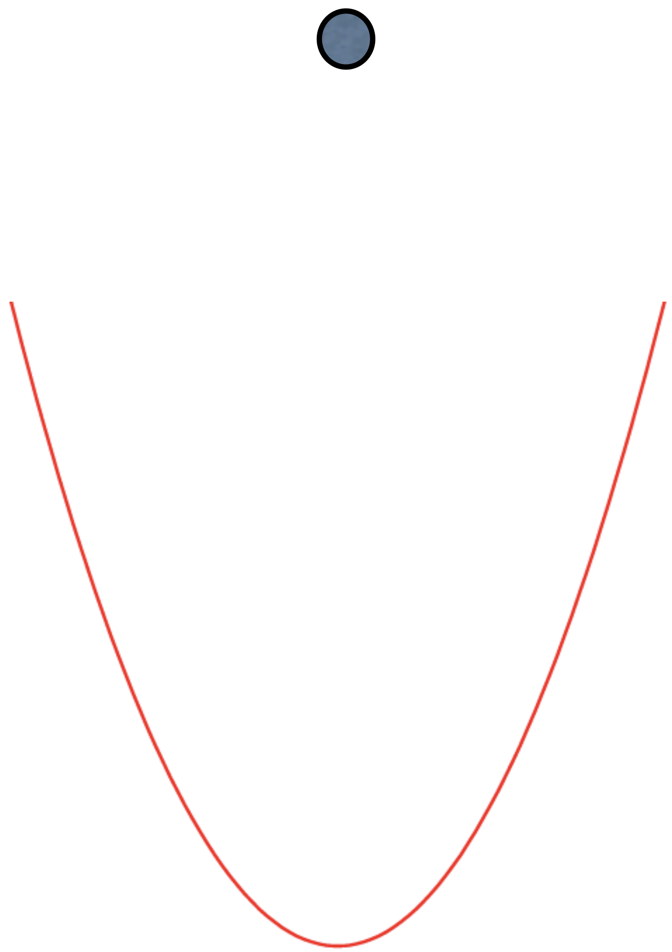
Slow - adiabatic



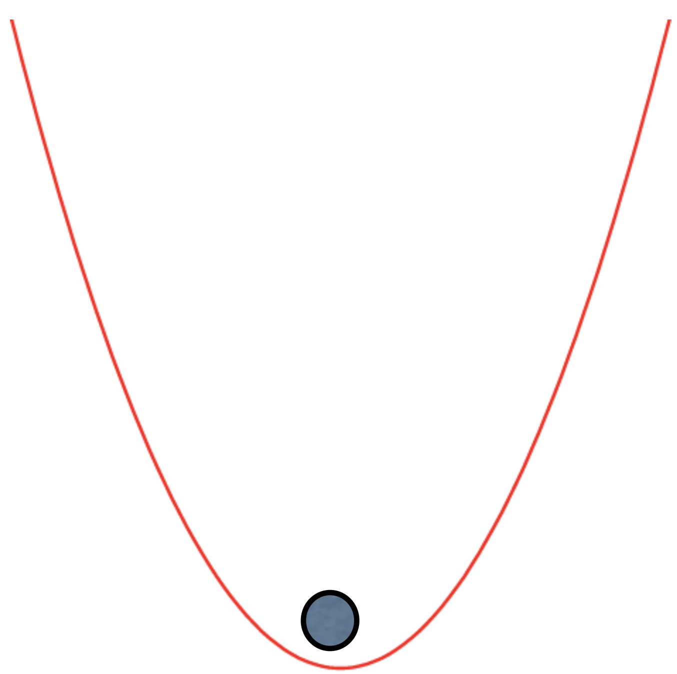
Fast - excitation



Slow - adiabatic



Fast - excitation



Slow - adiabatic

$$b_{\min} \approx \frac{\hbar}{m_e v}, \qquad b_{\max} \approx \frac{v}{3\omega_0} \implies \frac{b_{\max}}{b_{\min}} \approx \frac{m_e v^2}{2\hbar\omega_0}$$

$$\lambda \approx \ln\left(\frac{m_e v^2}{I}\right)$$

So in a neutral medium the Coulomb logarithm is basically the log of the ratio of the maximum possible energy transfer to the typical ionization energy of the neutral atoms.

For fast particles in an ionized medium, limits imposed by plasma frequency cutoff and thermal energy of the electrons.

No electromagnetic waves below the plasma frequency,

$$\omega_p = \left(\frac{n_e e^2}{\epsilon_0 m_e}\right)^{1/2} \qquad \frac{b_{\rm max}}{v} \approx \omega_p$$

electrons are not at rest if

$$\Delta E \approx kT_e$$

Classical Bethe-Bloch theory for energy loss of a charged ion moving through a medium

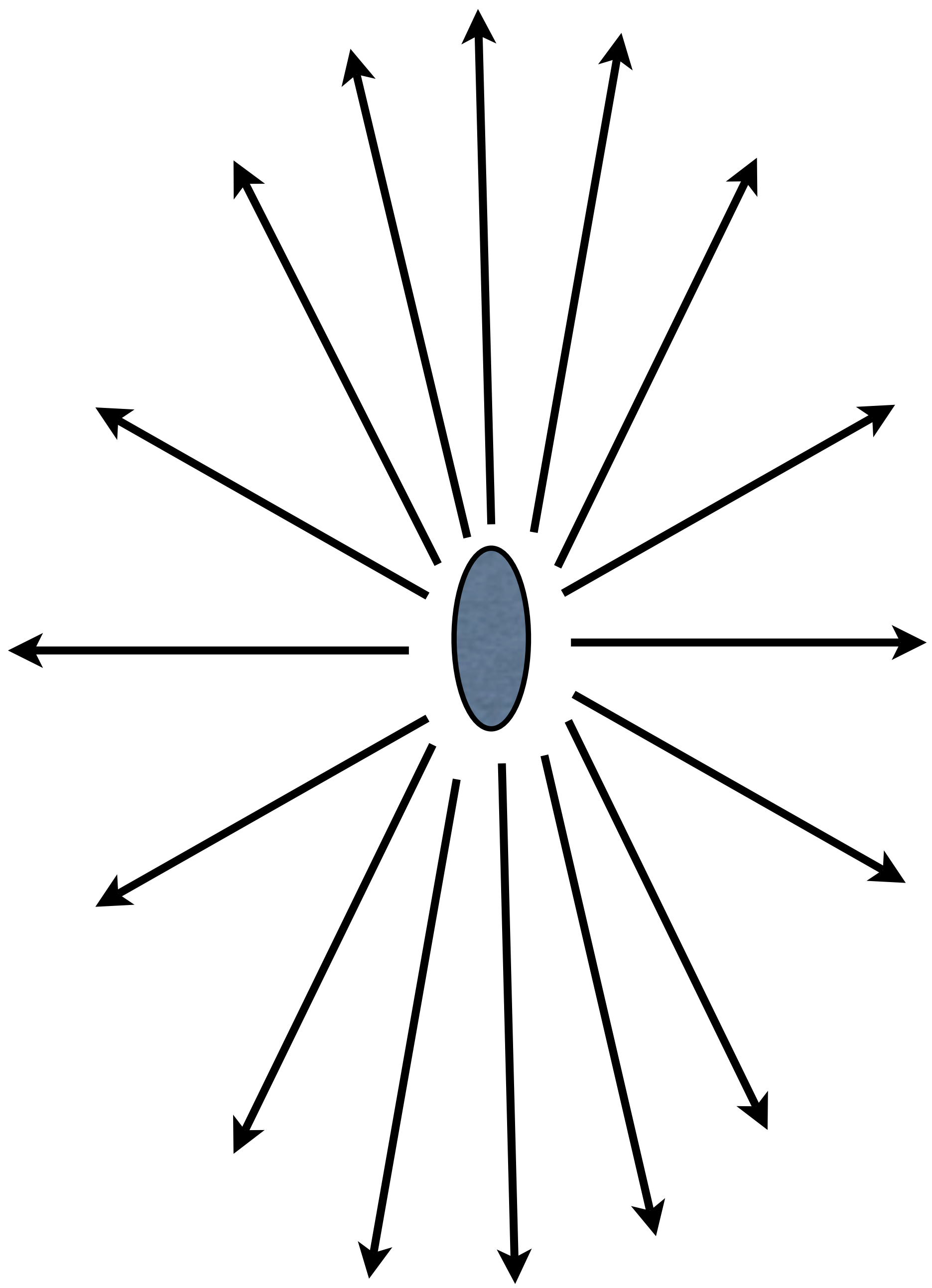
$$\frac{dE}{dx} = \frac{1}{v}\frac{d\,E}{d\,t} = -\frac{n_e Z^2}{m_e v^2}\left(\frac{e^4}{4\pi\epsilon_0^2}\right)\lambda$$

NB quadratic dependence on ion charge (need to worry about electron pick-up for very slow ions) and rapid increase of energy loss as ion slows.

$$\lambda \approx \left[\ln\left(\frac{2\gamma^2 m_e v^2}{\bar{I}}\right) - \frac{v^2}{c^2}\right]$$

Exercise - do the relativistic case!

Relativistic transformation of Coulomb field



electron 'feels' a pulse compressed by a factor of gamma in time and increased by a factor gamma in intensity.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

“Kick” to electron remains constant at fixed impact parameter.

$$\Delta t \rightarrow \frac{\Delta t}{\gamma}$$

$$E \rightarrow \gamma E$$

but minimum impact parameter decreases leading to slow logarithmic increase in ionization.

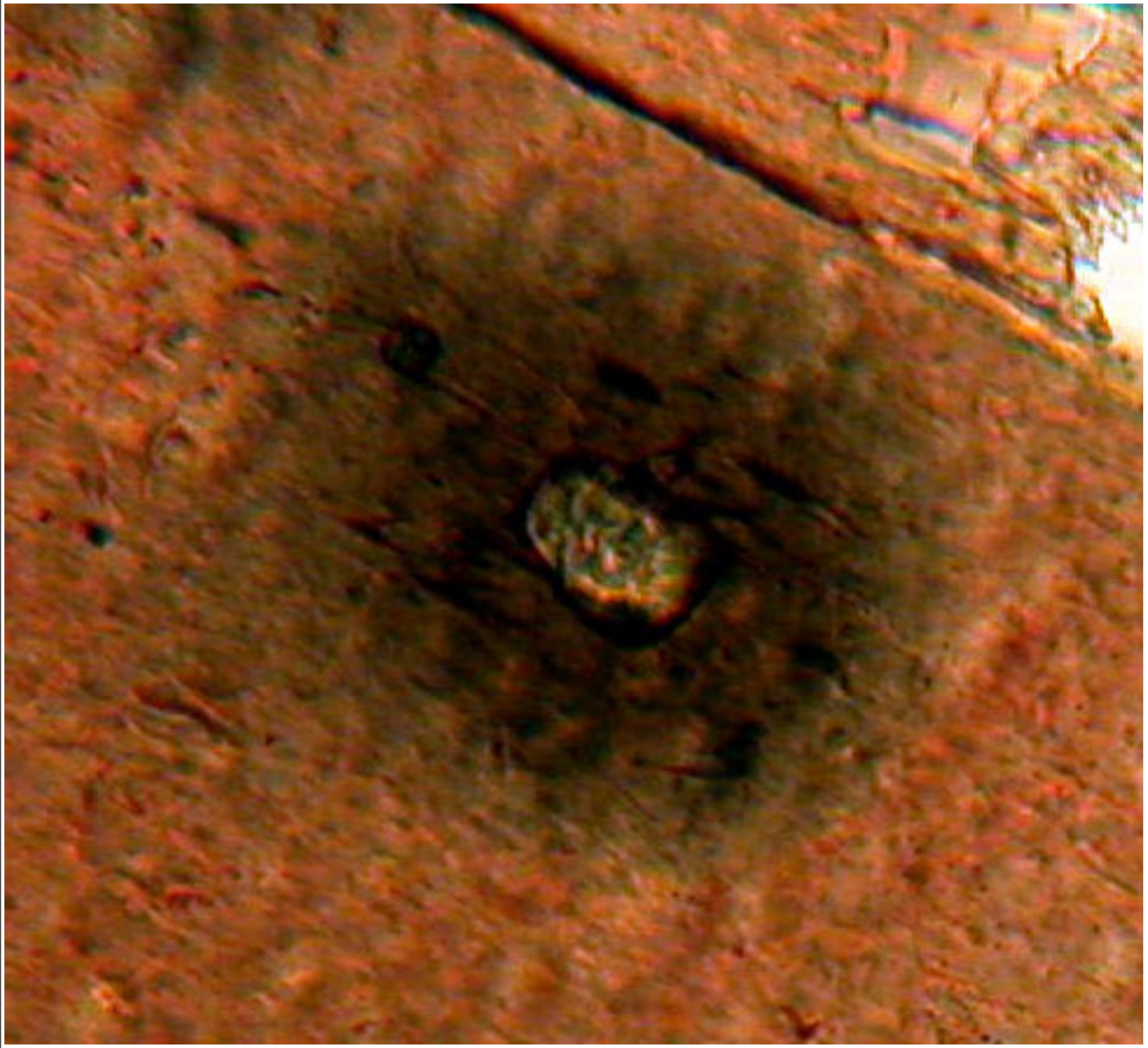
Leads to classical range-energy relations, ions of a given charge and energy travel a rather sharply defined distance before stopping.

$$\frac{dE}{dx} = -\frac{\alpha}{E}$$

$$EdE = -\alpha dx$$

$$E^2 - E_0^2 = -2\alpha(x - x_0)$$

Ion stops at range $E_0^2/2\alpha$



Explanation for pleochroic halos observed in many minerals around radioactive inclusions.

Also use of heavy ion beams for radiotherapy of cancer.

Energy loss is a minimum for mildly relativistic particles (slow logarithmic rise for highly relativistic particles, scales inversely with energy for non-relativistic particles).

$$\frac{dE}{dt} = -\frac{n_e Z^2}{m_e v}\left(\frac{e^4}{4\pi\epsilon_0^2}\right)\lambda$$

Exercise: put in numbers for a GeV proton in the ISM and show that the energy loss time scale is of order

$$E/\dot{E} \approx 10^8\,\mathrm{yr}$$

As originally pointed out by Fermi this means that if one can make relativistic protons, they can easily survive for very long times in the Galaxy, and in the low density ICM even for a Hubble time.

The nuclear collision time scale is also long. The radius of an atomic nucleus is rather accurately given by the simple formula (almost constant density of nuclear matter)

$$R = 1.2 \times 10^{-15} A^{1/3}\,\mathrm{m}$$

and the cross-sections are essentially geometric.

Thus for example the p-p inelastic cross section is of order

$$\sigma_{\rm pp} \approx 2.5 \times 10^{-30}\,{\rm m}^2$$

giving a collisional life time in a medium of density

$$n \approx 10^6\,{\rm m}^{-3} = 1\,{\rm cm}^{-3}$$

of about

$$t \approx \frac{1}{n\sigma_{\rm pp}c} \approx 3 \times 10^7\,{\rm yr}$$

Radiative losses of electrons

- Bremsstrahlung
- Synchrotron
- Inverse Compton

Life is harder for electrons!

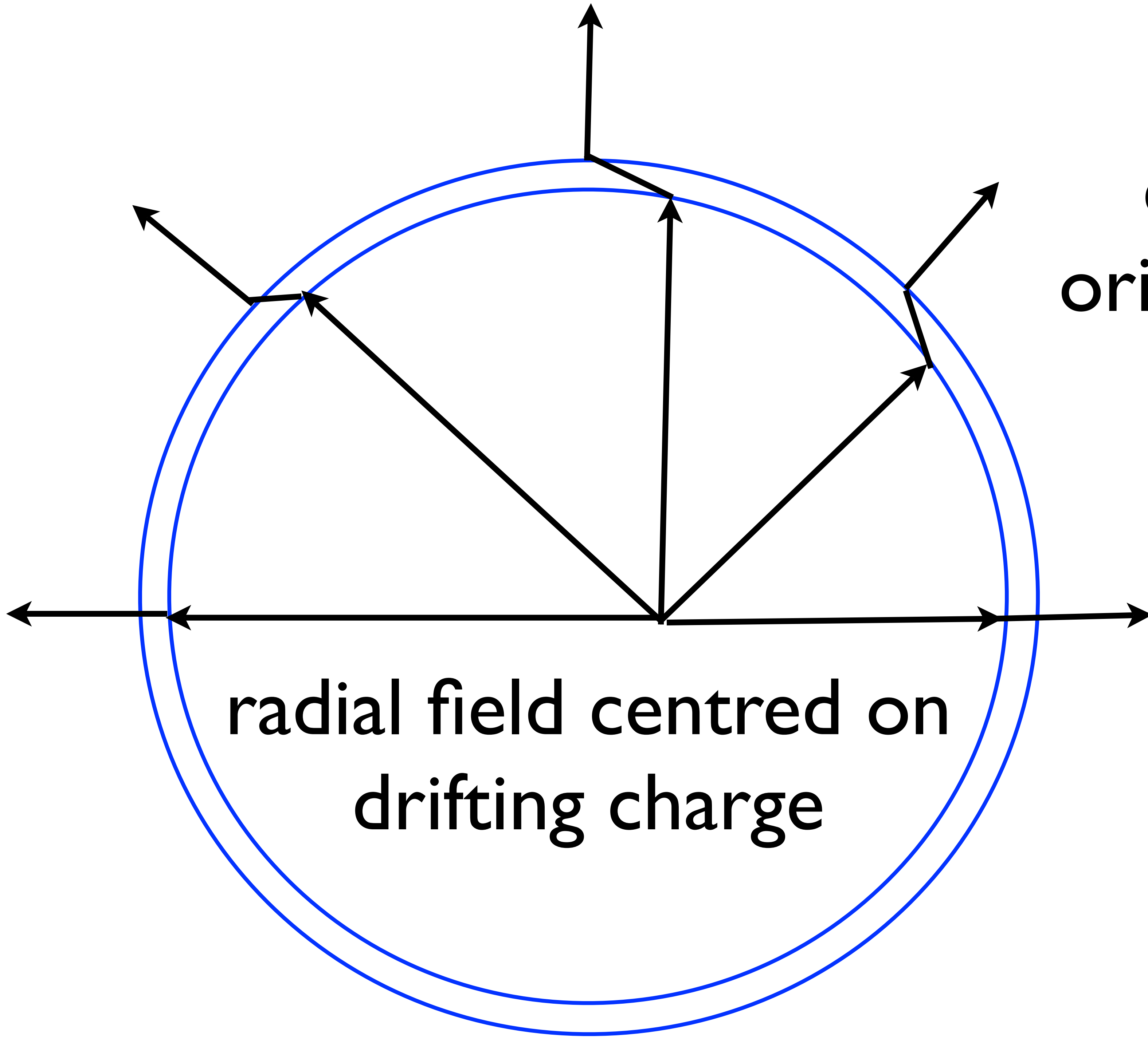
Accelerated charges radiate electromagnetic energy.

(NB uniformly moving charges do not radiate in a vacuum, but can do so in dielectric media, these are the important Cherenkov radiation and transition radiation effects)

Single most important formula in radiation theory is Larmor's expression for the energy radiated,

$$\frac{dE}{dt} = \frac{e^2\,|\ddot{x}|^2}{6\pi\epsilon_0 c^3}$$

Heuristic derivation



Charge initially at rest, given small impulse ΔV

Fields have to match across a thin spherical shell of thickness $c\Delta t$ and radius ct

The only way to do this is to have a tangential electric field in the shell of magnitude

$$
\begin{aligned}
E_{\mathrm{T}} &= \frac{t\Delta V}{c\Delta t} E_{\mathrm{R}} \sin\theta \\
&= \frac{r}{c^2} \frac{\Delta V}{\Delta t} \frac{e}{4\pi\epsilon_0 r^2} \sin\theta
\end{aligned}
$$

Now calculate energy content of this radiation field

$$
\begin{aligned}
\Delta\mathcal{E}_{\mathrm{E}} &= \int r^2 c\Delta t \frac{\epsilon_0}{2} E_{\mathrm{T}}^2 d\Omega \\
&= \frac{\epsilon_0}{c^3}\left(\frac{\Delta V}{\Delta t}\right)^2 \left(\frac{e}{4\pi\epsilon_0}\right)^2 \Delta t \int_0^\pi \pi \sin^3\theta d\theta \\
&= \frac{1}{c^3}\frac{e^2}{4\pi\epsilon_0}\left(\frac{\Delta V}{\Delta t}\right)^2 \frac{\Delta t}{3}
\end{aligned}
$$

and because this is a radiation field there is an equal amount of magnetic energy,

$$
\Delta\mathcal{E} = \Delta\mathcal{E}_{\mathrm{E}} + \Delta\mathcal{E}_{\mathrm{B}} = \frac{1}{c^3}\frac{e^2}{6\pi\epsilon_0}\left(\frac{\Delta V}{\Delta t}\right)^2 \Delta t
$$

So total energy radiated is just

$$\frac{\Delta\mathcal{E}}{\Delta t} = \frac{1}{c^3}\frac{e^2}{6\pi\epsilon_0}\left(\frac{\Delta V}{\Delta t}\right)^2$$

One reason this formula is so useful is that the power is a relativistic invariant,

$$\frac{\Delta\mathcal{E}}{\Delta t} = \frac{\Delta\mathcal{E}'}{\Delta t'}$$

First application, Thompson scattering of an EM wave by a free electron.

Sinusoidal E field incident on free electron

$$m_e \ddot{x} = eE_0 \sin(\omega t)$$

Accelerated charge radiates

$$\frac{\Delta \mathcal{E}}{\Delta t} = \frac{1}{c^3} \frac{e^2}{6\pi\epsilon_0} \left(\frac{e}{m_e} E_0 \sin(\omega t) \right)^2$$

$$= \frac{e^4}{12\pi\epsilon_0 c^3 m_e^2} E_0^2$$

Incident energy flux in EM wave is

$$\frac{1}{2}\epsilon_0 E_0^2 c$$

Scattering X-section is scattered/incident

$$\sigma_{\rm T} = \frac{e^4}{12\pi\epsilon_0 c^3 m_e^2}\frac{2}{\epsilon_0 c}$$
$$= \frac{e^4}{6\pi\epsilon_0^2 m_e^2 c^4}$$

Thompson cross-section for low-energy photon scattering on electrons! Note achromatic.

(Low energy means $h\nu \ll m_e c^2$).

Second application, Eddington luminosity

Radiation pressure on electrons

$$\frac{L}{4\pi r^2 c}\sigma_{\rm T}$$

Gravitational attraction on protons

$$\frac{GMm_p}{r^2}$$

Balance at

$$L_{\rm Edd} = \frac{4\pi G M m_p c}{\sigma_{\rm T}}$$

Bremsstrahlung

Fast electron flies past heavy nucleus and emits radiation as a result of the momentum kick it receives.

Usual impulse approximation trick:

$$m_e \ddot{x} = \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{b^2 + v^2 t^2}$$

Energy radiated in one collision at impact parameter b is

$$\Delta\mathcal{E} = \int \frac{1}{c^3}\frac{e^2}{6\pi\epsilon_0}\left[\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{1}{b^2+v^2t^2}\right]^2 dt$$

$$\approx \frac{Z^2e^6}{96\pi^3\epsilon_0^3c^3m_e^2}b^{-4}\frac{b}{v}$$

$$\propto Z^2b^{-3}v^{-1}$$

Integrating over collisions then gives

$$\frac{\Delta\mathcal{E}}{\Delta t} \approx \frac{Z^2e^6}{96\pi^3\epsilon_0^3c^3m_e^2}\int\frac{1}{b^3v}\,n_en_iv2\pi b\,db$$

$$= \frac{n_en_iZ^2e^6}{48\pi^2\epsilon_0^3c^3m_e^2}b_{\min}^{-1}$$

Key points

- Z^2 dependence - heavy ions very effective coolants of a plasma.
- Totally dominated by few close encounters
- Radiated field is precisely that described in the heuristic derivation of Larmor's formula, basically a short sharp pulse - in Fourier space flat spectrum up to cut-off
- Exact formulae use "Gaunt factor" corrections to these simple models.

Thermal Bremsstrahlung

$$\frac{1}{2}m_e v^2 = \frac{3}{2}kT_e$$

$$\Longrightarrow v = \sqrt{\frac{3kT_e}{m_e}}$$

Quantum limit

$$\frac{\Delta\mathcal{E}}{\Delta t} \approx \frac{n_e n_i Z^2 e^6}{48\pi^2\epsilon_0^3 c^3 m_e^2}\frac{m_e v}{\hbar} \propto T^{1/2}$$

Dominant cooling in very hot (coronal) plasmas

Note inefficient cooling at high temperatures,

$$t_{\rm cool} = \frac{\mathcal{E}}{\dot{\mathcal{E}}} \propto T^{1/2}$$

Classical limit can occur in cooler plasmas, but then usually dominated by line cooling

$$b_{\rm min} = \frac{Ze^2}{2\pi\epsilon_0 m_e v^2}$$

$$\frac{\Delta\mathcal{E}}{\Delta t} \approx \frac{n_e n_i Z^2 e^6}{48\pi^2\epsilon_0^3 c^3 m_e^2}\frac{2\pi\epsilon_0 m_e v^2}{Ze^2} \propto T$$

Relativistic Bremsstrahlung

Relativistic electron flies past heavy ion/atom

Transform to frame where electron is stationary, evaluate instantaneous acceleration and resulting radiation, then transform back to observer frame.

“Kick” is independent of Lorentz factor and depends only on impact parameter, but interaction period is shorter.

Maximum photon energy that can be produced is entire energy of the electron, so in observer (lab) frame

$$h\nu \leq \gamma m_e c^2$$

In the electron rest frame this implies

$$h\nu \leq m_e c^2$$

which is in any case the condition not to have to include full QED effects.

Thus get a more restrictive quantum limit on $b_{\min}$

Further complications arise in neutral media due to shielding of the nuclear charge by bound atomic electrons - see Longair.

Bottom line is that non-thermal Bremsstrahlung produces spectra extending up to the maximum electron energy and the spectral slope is essentially that of the primary electron spectrum.

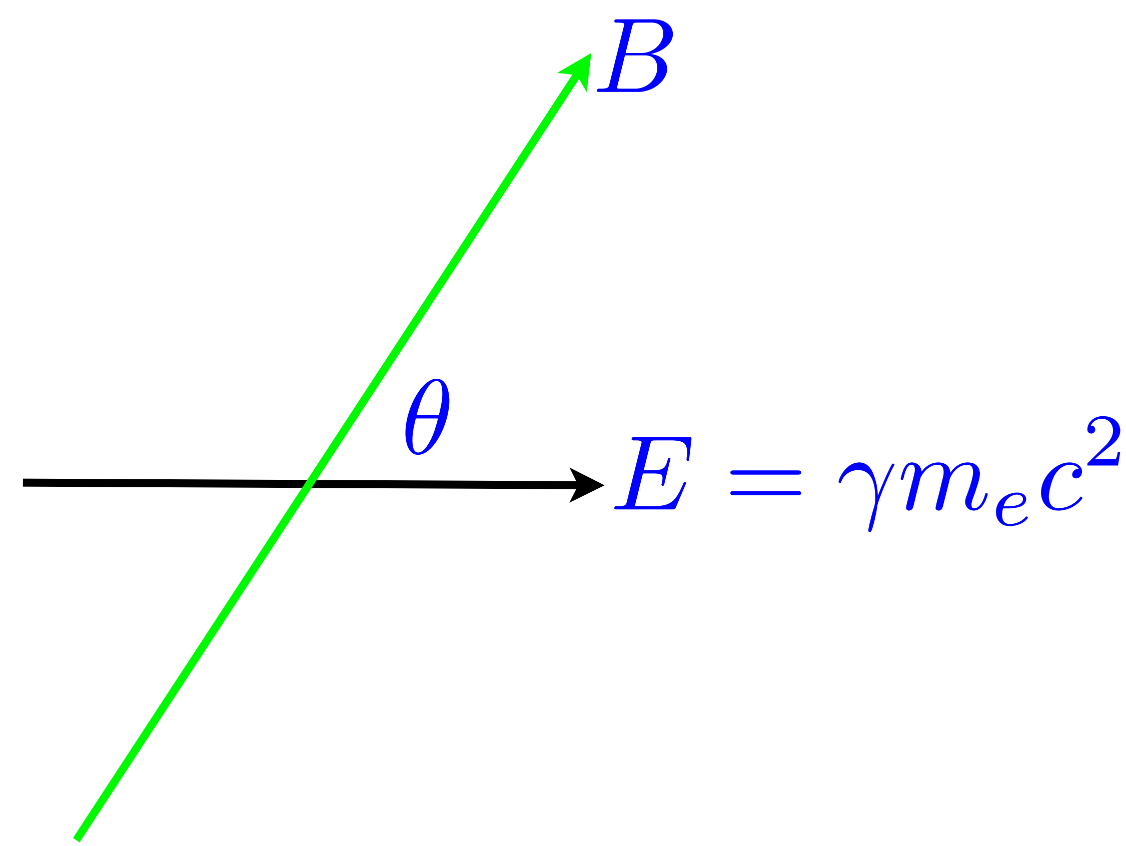
Can be treated as Compton scattering of virtual photons from the field of the ion.

Synchrotron radiation

- Also called magnetobremsstrahlung
- Radiation from relativistic electrons in a magnetic field
- Major problem for lepton accelerators (terrestrial and cosmic)
- Main source for radio astronomy, but also optical emission of the Crab nebula, X-ray

Consider relativistic electron in uniform magnetic field inclined at angle θ to electron direction.

Observer (lab) frame



$$\begin{aligned} B_{\parallel} &= B\cos\theta \\ B_{\perp} &= B\sin\theta \end{aligned}$$

Do a Lorentz transformation to electron rest frame.

$$
\begin{aligned}
B'_{\parallel} &= B_{\parallel} \\
B'_{\perp} &= \gamma(B_{\perp} - v \times E) = \gamma B_{\perp} \\
E'_{\parallel} &= E_{\parallel} = 0 \\
E'_{\perp} &= \gamma(E_{\perp} + v \times B) = \gamma v \times B_{\perp}
\end{aligned}
$$

In its instantaneous rest frame only force on the electron comes from induced electric field

$$\ddot{x} = \frac{e}{m_e}\gamma v B \sin\theta$$

Thus radiates at rate

$$\dot{\mathcal{E}}' = \dot{\mathcal{E}} = \frac{e^2}{6\pi\epsilon_0 c^3} |\ddot{x}|^2$$

$$= \frac{e^2\gamma^2 B^2 \sin^2\theta v^2}{6\pi\epsilon_0 c^3 m_e^2}$$

$$= 2\sigma_T \left(\frac{v}{c}\right)^2 c \frac{B^2}{2\mu_0} \sin^2\theta$$

(need to use $c^2 = 1/(\mu_0\epsilon_0)$)

For highly relativistic particles with

$$v = c$$

can write this as

$$\dot{\mathcal{E}} = 2\sigma_T c U_{\mathrm{mag}} \sin^2\theta$$

and if we average over an isotropic distribution

$$\dot{\mathcal{E}} = \frac{4}{3}\sigma_T c U_{\mathrm{mag}}$$

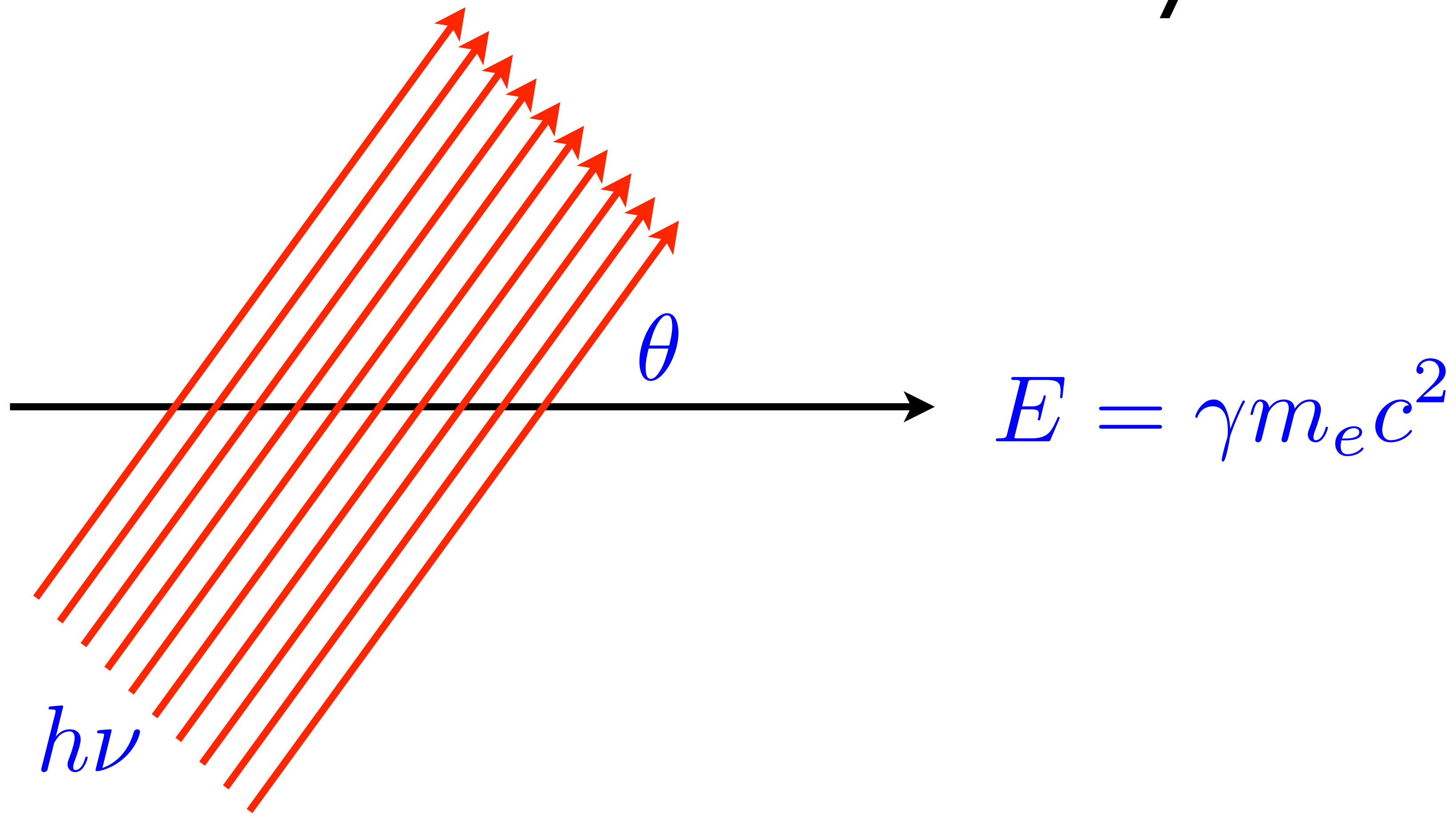
Can interpret this as Compton scattering of virtual photons from the magnetic field.

Acquires additional significance and usefulness from a related result in Inverse Compton scattering.

Will not discuss spectrum, beaming, polarization etc here - see standard textbooks.

Inverse Compton Scattering

As before, consider an ultra-relativistic electron with Lorentz factor γ, but this time travelling in a photon field of number density n.



Need to transform to
electron rest frame as usual

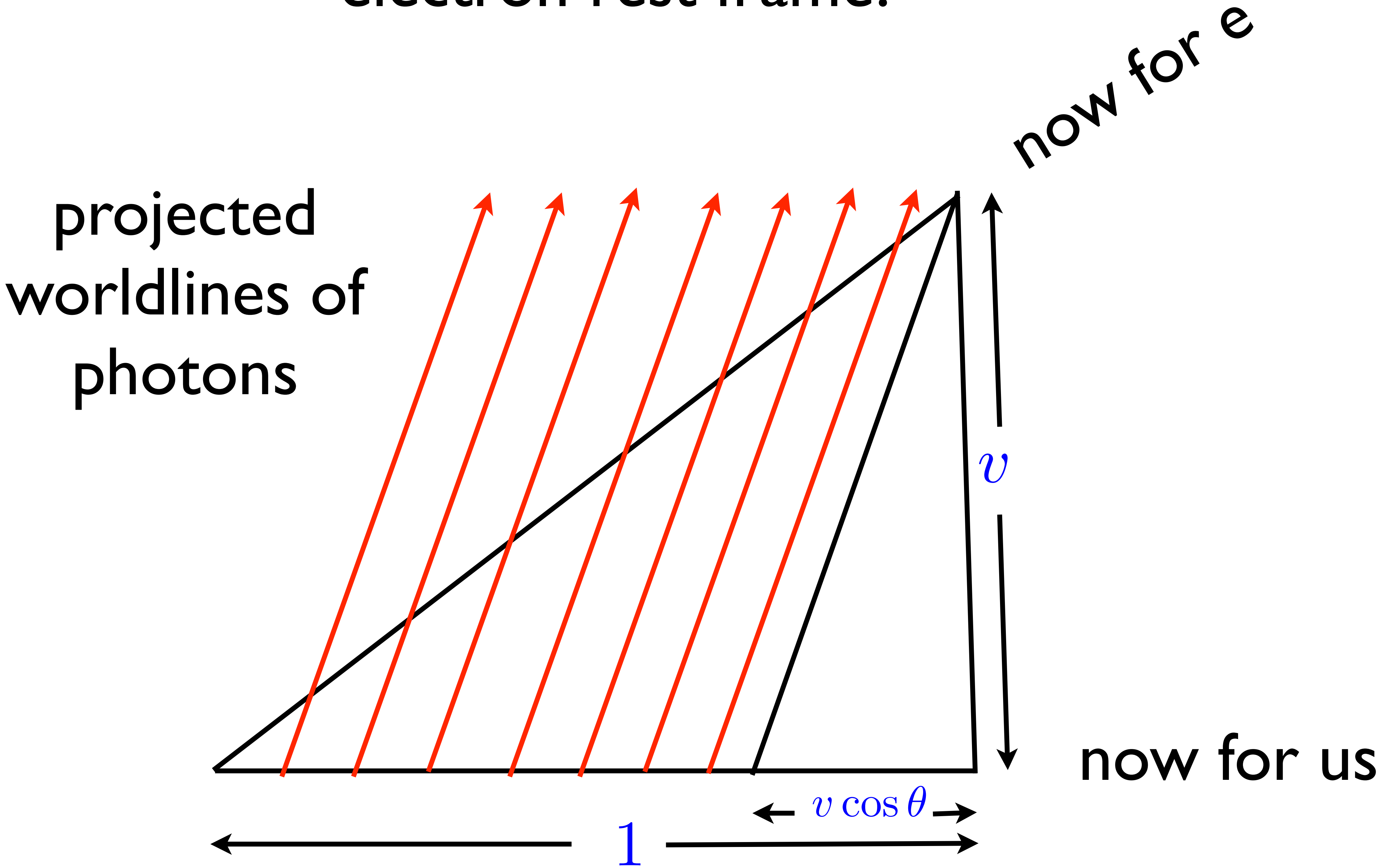
In lab frame 4 energy momentum vector of a photon is just

$$h\nu\,(1, \cos\theta, \sin\theta, 0)$$

and in the electron frame it is

$$\gamma h\nu\left(1 - v\cos\theta, \cos\theta - v, \frac{\sin\theta}{\gamma}, 0\right)$$

Need flux of the boosted photons in electron rest frame.



$$n'\sqrt{1-v^2} = n(1 - v\cos\theta)$$

Energy flux in electron frame is thus

$$\gamma^2(1 - v\cos\theta)^2 nh\nu c$$

and scattered power is thus

$$\dot{\mathcal{E}} = \sigma_{\mathrm{T}}\gamma^2(1 - v\cos\theta)^2 nh\nu c$$

for relativistic particles and averaging over angle

$$\begin{aligned} \langle(1-\cos\theta)^2\rangle &= \frac{1}{2}\int_{-1}^{+1}(1-\mu)^2 d\mu \\ &= 1 + \frac{1}{3} = \frac{4}{3} \end{aligned}$$

Thus scattered IC power is just

$$\frac{4}{3}\sigma_{\mathrm{T}} c\, \mathcal{E}_{\mathrm{rad}} \gamma^2$$

Exactly the same as for synchrotron!

$$\frac{L_{\mathrm{IC}}}{L_{\mathrm{Sync}}} = \frac{\mathcal{E}_{\mathrm{rad}}}{\mathcal{E}_{\mathrm{Mag}}}$$

Note that each photon gets two boost factors, giving a gamma squared energy gain - very effective.

Photon energy after first boost must be less than the electron rest mass, otherwise Thompson scattering has to be replaced by Klein-Nishina

$$\begin{aligned} \gamma h\nu &\ll m_e c^2 \\ \Longrightarrow \gamma &\ll \frac{m_e c^2}{h\nu} \\ \gamma^2 h\nu &\ll \frac{(m_e c^2)^2}{h\nu} \end{aligned}$$