Analytical View on Diffusive and Convective Cosmic Ray Transport in Elliptical Galaxies

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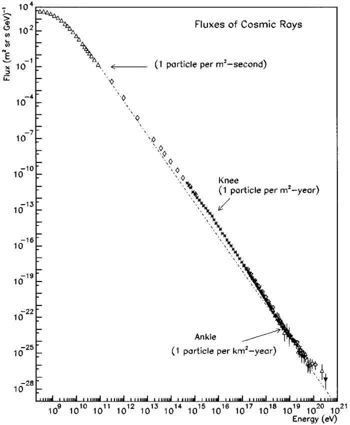
Outline

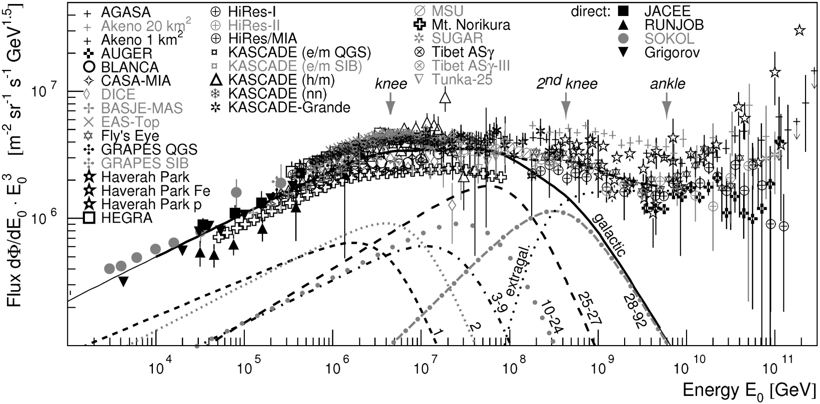
1. Motivation
2. Theory
3. Methods
4. Results
5. Conclusion

Cosmic Ray Spectrum (1)

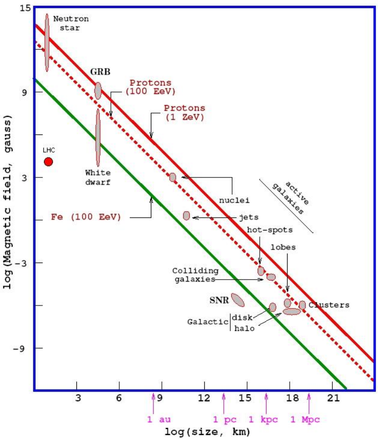


- Cosmic Rays (CRs) discovered by Viktor Hess (1912)
- Consisting mostly of protons, but also of heavier charged atomic nuclei
- Nearly perfect power law with spectral index -2.7
- Features like the "Knee" and the "Ankle" theoretically still unknown

Cosmic Ray Spectrum (2)

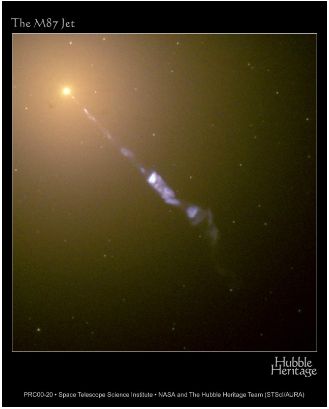


The "Hillas" Plot



- Energy dependence of Cosmic Ray sources
- Most likely high energy Cosmic Ray sources: Gamma Ray Bursts (GRB), Active Galaxies (AG), galaxy mergers
- Nearby AG may be the main source of ultra-high energy CRs

The Giant Elliptical Galaxy M87



- Nearby elliptical radio galaxy (d$\approx$ 60 Mly, r$\approx$ 60 kly)
- Dominant galaxy of the Virgo cluster, type E1, mass$\approx 10^{11}$ solar masses
- Active Galactic Nucleus (AGN), big jet from accretion onto a black hole (mass $\approx 3 * 10^9$ solar masses)
- Particle acceleration within the jet due to Fermi I processes.

Kinetic Theory of Astrophysical Plasmas

Starting point is the equation for the time-evolution of the phase space density $f_a(\mathbf{x}, \mathbf{p}, t)$ of particle species a:

The Relativistic Vlasov Equation

$$\frac{\partial f_a}{\partial t} + \mathbf{v}\frac{\partial f_a}{\partial \mathbf{x}} + \dot{\mathbf{p}}\frac{\partial f_a}{\partial \mathbf{p}} = S_a(\mathbf{x}, \mathbf{p}, t) \tag{1}$$

with the equations of motion

$$\dot{\mathbf{p}} = q_a \left[\mathbf{E}(\mathbf{x}, t) + \frac{\mathbf{v} \times \mathbf{B}(\mathbf{x}, t)}{c} \right]$$

and

$$\dot{\mathbf{x}} = \mathbf{v} = \frac{\mathbf{p}}{\gamma m_a}$$

Generalised Diffusion-Convection Equation

In quasilinear theory and the "Diffusion approximation" one finds a generalised diffusion-convection equation for Cosmic Ray protons

The "Leaky Box" Equation

$$\frac{\partial f}{\partial t} - S(\mathbf{r}, p, t) = \frac{\partial}{\partial z}\left(\kappa_{zz}\frac{\partial f}{\partial z}\right) + \\ + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 A \frac{\partial f}{\partial p} + \frac{p^3}{3}\frac{\partial V}{\partial z} f - p^2 \dot{p} f\right) - \\ - \frac{f}{T_c(z, p)} \qquad (2)$$

- Right hand side involves all physically relevant processes

Generalised Diffusion-Convection Equation

For an analytical solution the steady state problem has to be solved:

$$\mathcal{L}_{\mathbf{r}} f(\mathbf{r}, p) + \mathcal{L}_p f(\mathbf{r}, p) = -S(\mathbf{r}, p) \tag{3}$$

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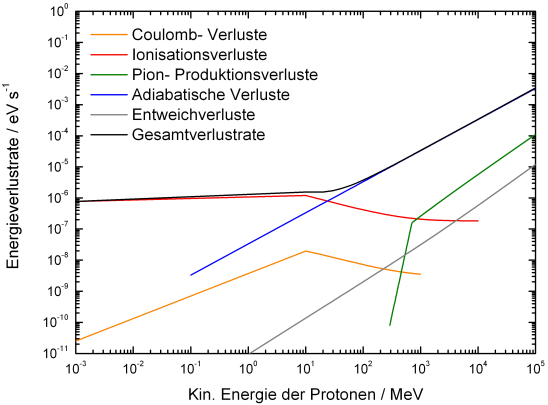
- Momentum operator describes momentum diffusion and convection

$$\mathcal{L}_p f(\mathbf{r}, p) \equiv \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 A \frac{\partial f}{\partial p} + \frac{p^3}{3} \nabla \mathbf{V} f - p^2 \dot{p} f \right) - \frac{f}{T_c(z, p)}$$

Physically Relevant Energy Loss Processes in M87



Separation of Spatial and Momentum Problem

Important class of exact analytical solution of the transport equation (eq.(3)) possible, if

$$\mathcal{L}_{\mathbf{x}}(\mathbf{x}, p) = g(p)\mathcal{O}_{\mathbf{x}}(\mathbf{x}) \quad ; \quad \mathcal{L}_p(\mathbf{x}, p) = h(\mathbf{x})\mathcal{O}_p(p)$$

The source function should be a product of two seperable functions, i.e.

$$S_a(\mathbf{x}, p) = Q_1(\mathbf{x})Q_2(p)$$

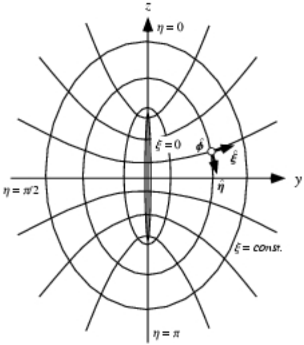
The following convolution gives the

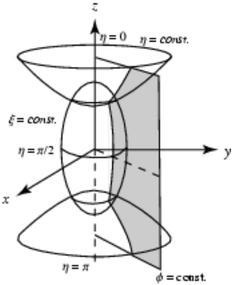
Formal Mathmatical Solution

$$f_a(\mathbf{x}, p) = \int_0^\infty du G(p, u) P(\mathbf{x}, u) \tag{4}$$

Special Spatial Geometry

Prolate spheroidal coordinates incorporated in spatial problem:





Formal Analytical Solutions

- **Spatial Problem:**
  Solved by separation of variables by an Eigenfunction expansion (solution functions and weighting coefficients) in terms of modified Besselfunctions and Legendre-polynomials. Weighting coefficients are adjusted to the spatial boundary conditions. (For analytical solution "Free Escape" boundary conditions necessary)

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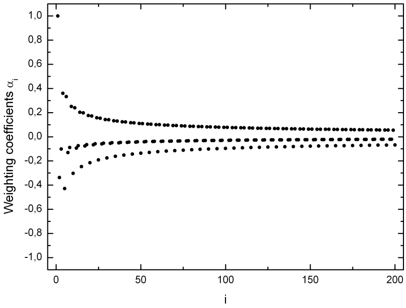
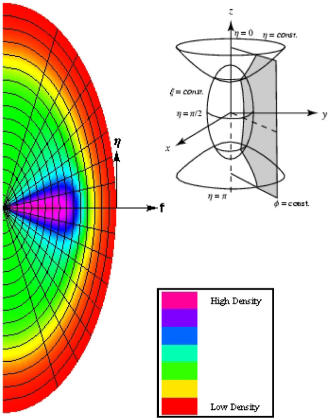
- **Momentum Problem:**
  Solved by standard technique (Greens function) in terms of confluent hypergeometric functions after finding the solution of the homogeneous Riccati-type differential equation. Momentum boundary conditions are finiteness at the boundaries $p = 0$ and $p = \infty$

Consistency Checks

- The solution matches given physical boundary conditions.
- The eigenfunction expansion with the weighting coefficients converge, the maximum number of expansion coefficients has to be adjusted to the needed accuracy.
- In the limit of small ellipticity the solution function satisfies the one for a sperical geometry found by Schlickeiser et. al. (1987).



Spatial Problem Solution



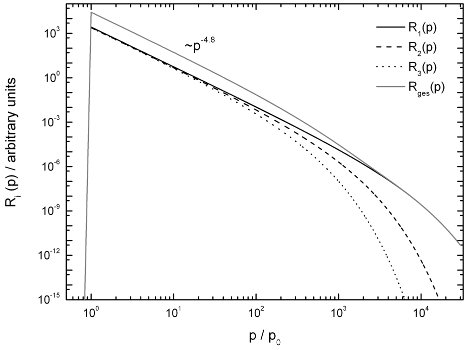
Proton density with jet as source

- Jet modelled via source function $Q_2(p) = \delta(\eta - \eta_{inj})\Theta(f - f_{max})$
- Normalized distribution function
- Yellow border defines edge of the galaxy

Results

- Jet gets broadened due to diffusion at large distance to the center
- Jet fills the whole galaxy with energetic particles

Momentum Problem Solution



- Because of eigenfunction expansion in spatial coordinates n momentum differential equations have to be solved
- Delta-shape momentum injection at $p = p_0$

Conclusion

- Analytical solution of the cosmic ray transport equation has been found
- Model extendable to all kind of elliptical galaxies
- Predictions of this model can be testet with measured physical parameters in the case of M87

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Further prospects:

- Calculation of the M87 proton-synchrotron spectrum and comparison with measured spectra
- Calculation of the proton spectrum leaving M87 and estimation of the contribution to the extragalactical cosmic ray background measured at earth

References

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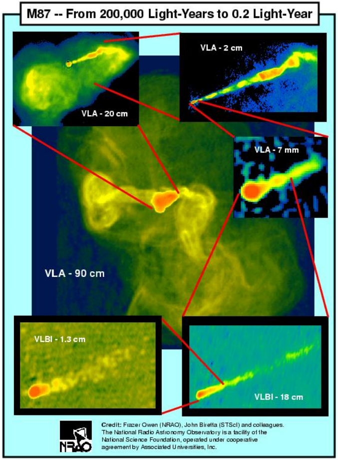
M87 in Radio Wavelength



- Radio images of M87 (one of the brightest radio sources in the sky) from HEGRA data (Owen et al. 2000)
- Synchrotron radiation of mainly electrons, but also protons
- Halo as well as the jet are visible
- Diffusion processes identificable

Physically Relevant Processes in Eq.(3)

Transport Operators

$$\mathcal{L}_{\mathbf{r}} f(\mathbf{r}, p) \equiv \nabla \left[\kappa(\mathbf{r}, p) \nabla f - \mathbf{V} f \right]$$

$$\mathcal{L}_p f(\mathbf{r}, p) \equiv \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 A \frac{\partial f}{\partial p} + \frac{p^3}{3} \nabla \mathbf{V} f - p^2 \dot{p} f \right) - \frac{f}{T_c(z, p)}$$

- Energy gain due to Fermi I/II processes in jet-shockfronts
- Adiabatic losses due to velocity gradients in the galactic wind
- Coulomb and ionisation losses of CRs in the ambient gas
- Pion production losses by proton-proton collisions
- Catastrophic losses by fragmentation of cosmic ray particles and by escape out of the galaxy

Formal Mathematical Solution

Eq.(4) is the formal mathematical solution, if $G(p, u)$ satisfies given momentum boundary conditions and

$$\frac{\partial G}{\partial u} = \frac{1}{g(p)} \mathcal{O}_p G \tag{5}$$

and $P(\mathbf{x}, u)$ satisfies given spatial boundary conditions and

$$\frac{\partial P}{\partial u} = \frac{1}{h(\mathbf{x})} \mathcal{O}_{\mathbf{x}} P \tag{6}$$

Spatial Problem

Here the spatial operator $\mathcal{O}_{\mathbf{x}}$ is of Sturm-Liouville type and so the solution can be expanded in the complete Eigenfunction system $H_i(\mathbf{x})$ of $\mathcal{O}_{\mathbf{x}}$:

$$P(\mathbf{x}, u) = \sum_i A_i(\mathbf{x}) e^{-\lambda_i^2 u} \tag{7}$$

with

$$A_i(\mathbf{x}) = c_i H_i(\mathbf{x})$$

The λ_i denote the spatial eigenvalues of $\mathcal{O}_{\mathbf{x}}$ and the expansion coefficients c_i weight the solution functions $H_i(\mathbf{x})$.

Momentum Problem

Inserting eq.(7) into the convolution leads to

$$M_a(\mathbf{x}, p) = \sum_i A_i(\mathbf{x}) R_i(p)$$

where each function

$$R_i(p) \equiv \int_0^\infty du G(p, u) e^{-\lambda_i^2 u}$$

obeys the ordinary differential equation

$$\mathcal{O}_p R_i(p) - \lambda_i^2 g(p) R_i(p) = -Q_2(p)$$