Particle acceleration in general relativistic environments

Schule fuer Astroteilchenphysik 4-12. 10. 2007

Plan of the talk

- Motivation
  - Kerr BH
  - Jet engine
- How 2do GRMHD
  - 3+1 Split
  - Formulation
  - Simulation
- GRPIC from the scratch
  - Motivation
  - Formulation
- Theory versus Observation

Morphology

$$ds^2 = -\alpha^2 dt^2 + \tilde{\omega}(d\phi - \omega dt)^2 + \rho^2 / \Delta \, dr^2 + \rho^2 d\theta^2$$

Boyer-Lindquist coordinates

where $\alpha = \frac{\rho\sqrt{\Delta}}{\Sigma}$ **lapse function (→ redshift)**

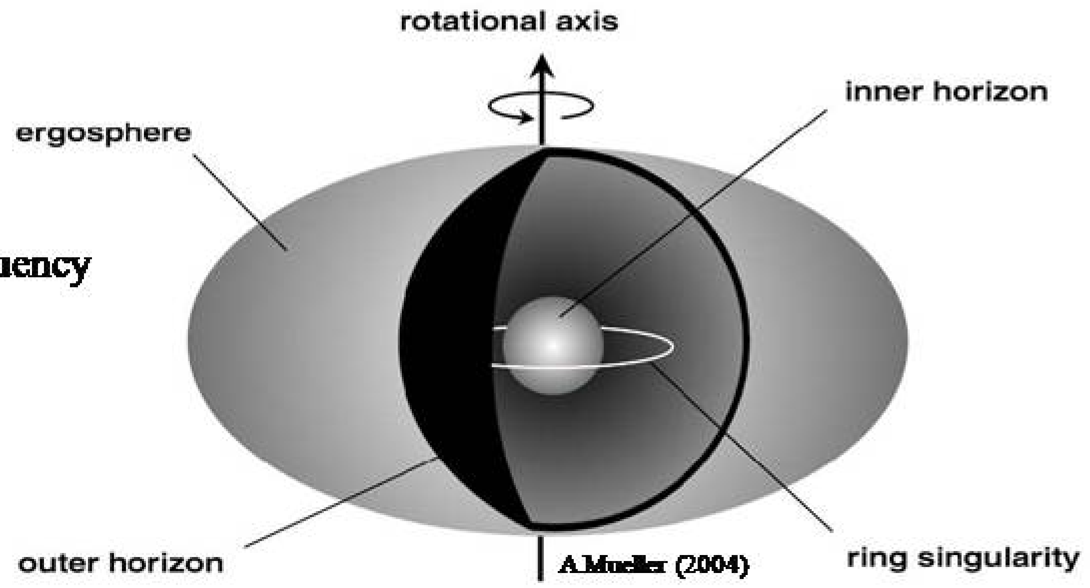
$$\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta$$

$$\Delta = r^2 - 2Mr + a^2$$

$$\rho^2 = r^2 + a^2 \cos^2\theta$$

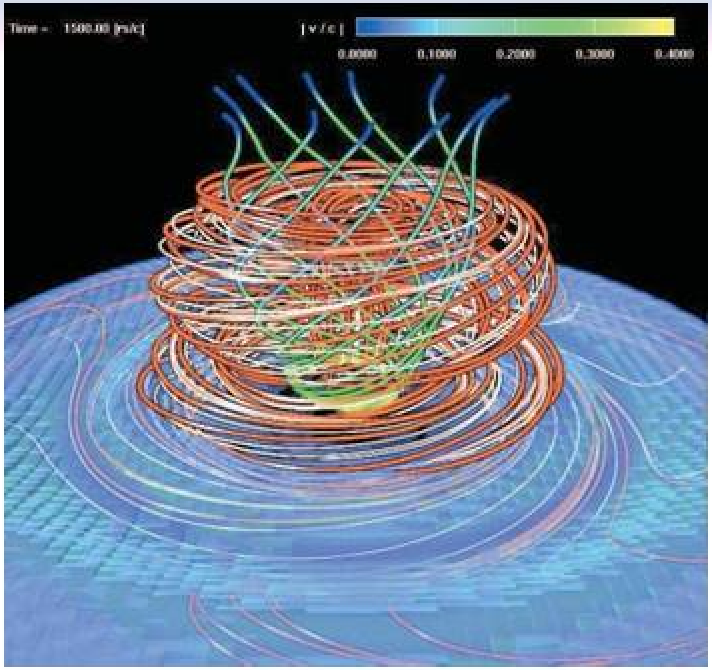
$\omega = \frac{2aMr}{\Sigma^2}$ **frame-dragging frequency**

$$\tilde{\omega} = \frac{\Sigma}{\rho} \sin\theta$$



Rotating BH are essential to explain jet formation!

Jet engine



(Kato et al. 2004)

Questions of Jet formation:

1. Relativistic outflows?
2. Collimation to a narrow outflow?

Blandford-Znajek process (1977)

- accretion feeds Kerr BH
- BH interacts with magnetic field
- pair production (leptonic plasma)
- extraction of rotational energy
- acceleration of leptonic plasma v ~ c
- works **only** with Kerr BH!
- energy source: BH

Blandford-Payne process (1982)

- accretion disk, magnetized
- plasma extraction along magnetic flux tubes
- magnetic pressure and centrifugally driven outflow v ~ v_kep
- works **without** BH
- energy source: disk

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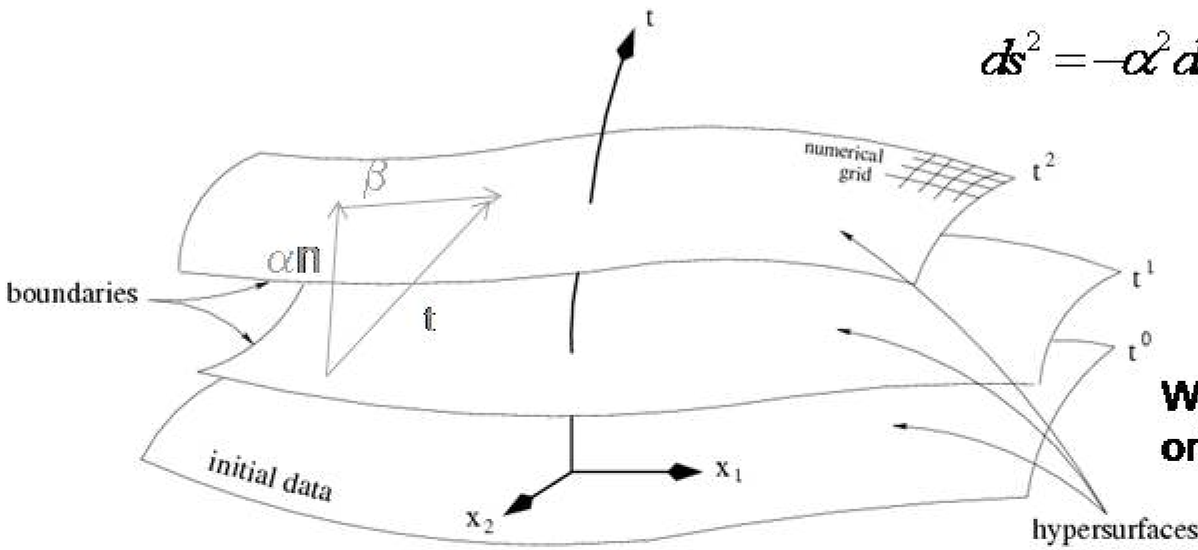
Doing GRMHD - One slide recipe

1. Specify space-time geometry (e.g. Kerr background)
2. Take a coordinate system: Kerr-Schild vs. Boyer-Lindquist
3. Choose some fluid -> energy-momentum tensor $T^{\mu\nu}$
4. Thin sandwich decomposition (3+1 split)
5. Take a solver : conservative/ non-conservative schemes
6. Specify initial and boundary conditions
7. Solve set of GRMHD equations

3+1 Split (ADM 1962)

Foliate the space-time with *t=const* spatial hypersurfaces Σ_t



$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

Where n is the unit time-like 4-vector orthogonal to Σ_t such that

$$n = \frac{1}{\alpha}(\partial_t - \beta^i \partial_i)$$

Eulerian observers measures: $v^i = \frac{1}{\alpha}\left(\frac{u^i}{u^t} + \beta^i\right)$ transport velocity

3+1 Split (ADM 1962)

electromagnetic field tensor $\quad F^{\mu\upsilon}$

U_υ 4-velocity of an arbitrary observer

$$E^\mu = F^{\mu\upsilon} U_\upsilon$$

$$B^\mu = {}^* F^{\mu\nu} U_\upsilon = \frac{1}{2} \varepsilon^{\mu\upsilon\kappa\lambda} U_\upsilon F_{\lambda\kappa}$$

Ohm's law

$$J^\mu + (J^\upsilon u_\upsilon) u^\mu = \sigma\, F^{\mu\upsilon} u_\upsilon$$

u_υ fluid's 4-velocity

Non-relativistic limit: $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Ideal MHD limit ($\sigma \rightarrow \infty$): $F^{\mu\upsilon} u_\upsilon = 0\,;\quad J \times B = 0$

$$\Rightarrow \quad E_{fluid}{}^\mu = 0\,; \qquad E^\mu = \frac{1}{W} \eta^{\mu\nu\lambda\delta} u_\nu U_\lambda B_\delta$$

3+1 Split (ADM 1962)

magnetic field measured by a comoving observer:

$$b^\mu \equiv \frac{1}{2\sqrt{4\pi}} \varepsilon^{\mu\upsilon\kappa\lambda} u_\upsilon F_{\lambda\kappa} = -\frac{P^\mu{}_\upsilon B^\upsilon;}{\sqrt{4\pi U_\lambda u^\lambda}} \qquad P_{\mu\upsilon} = g_{\mu\upsilon} + u_\mu u_\upsilon$$

$$T_{\text{em}}^{\mu\upsilon} = \frac{1}{4\pi}\left(F^{\mu\lambda}F^{\upsilon}{}_{\lambda} - \frac{1}{4} g^{\mu\upsilon} F_{\alpha\beta}F^{\alpha\beta}\right) = b^2 u^\mu u^\upsilon + \frac{1}{2} b^2 g^{\mu\upsilon} - b^\mu b^\upsilon$$

$$T_{fluid}^{\mu\upsilon} = \rho h u^\mu u^\upsilon + P g^{\mu\upsilon}$$

enthalpy: $h = 1 + \varepsilon + \frac{P}{\rho}$

EOS: $P = P(\rho, \varepsilon)$

ε = specific inertial energy density

ρ = rest mass density

$$T^{\mu\upsilon} = T_{fluid}^{\mu\upsilon} + T_{em}^{\mu\upsilon} = \left(\rho h + b^2\right) u^\mu u^\upsilon + \left(P + \frac{b^2}{2}\right) g^{\mu\upsilon} - b^\mu b^\upsilon$$

3+1 Split - GRMHD

1. Mass conservation $\nabla_\mu(\rho u^\mu) = 0$
2. Energy & momentum conservation $\nabla_\mu T^{\mu\nu} = 0$
3. Maxwell's equations $\nabla_\nu F^{\mu\nu} = 4\pi J^\mu , \quad \nabla_{[\mu}F_{\nu\lambda]} = 0$
   - Induction equation $\partial_t(\sqrt{\gamma}B^i) + \partial_j\left[\sqrt{\gamma}\left(v^j B^i - v^i B^j\right)\right] = 0$
   - no sinks/sources $\partial_j\left(\sqrt{\gamma}B^j\right) = 0$

Hyperbolic system, flux conserving.

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}\vec{u}}{\partial t} + \frac{\partial\sqrt{-g}\vec{f}^i}{\partial x^i}\right) = \vec{s}$$

+ constraint equation

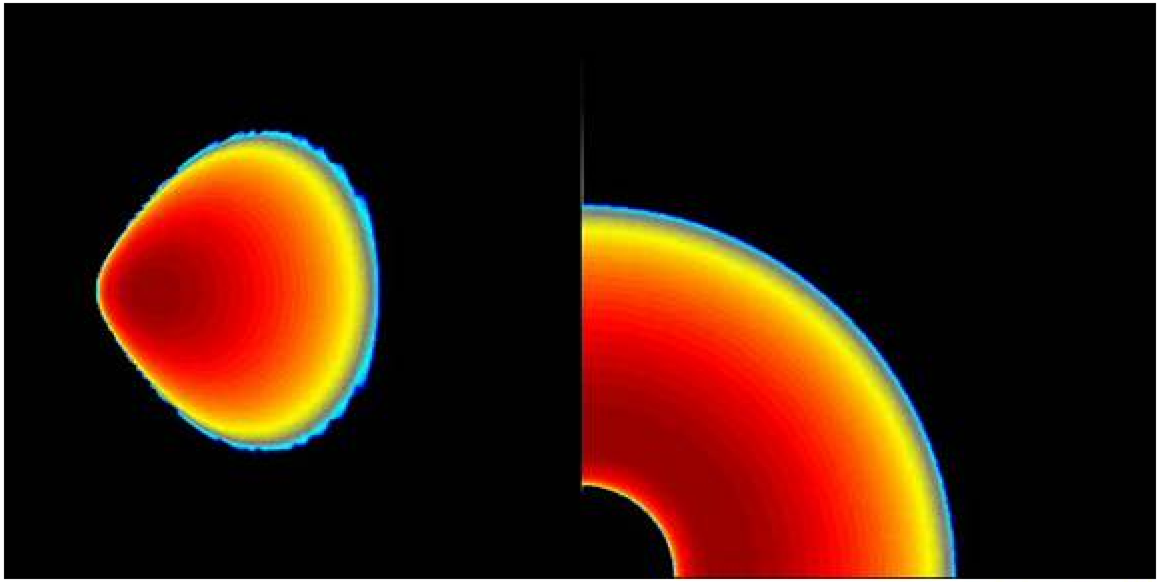
$\vec{u}$ state vector of CQ

$\vec{f}^i$ fluxes

$\vec{s}$ sources

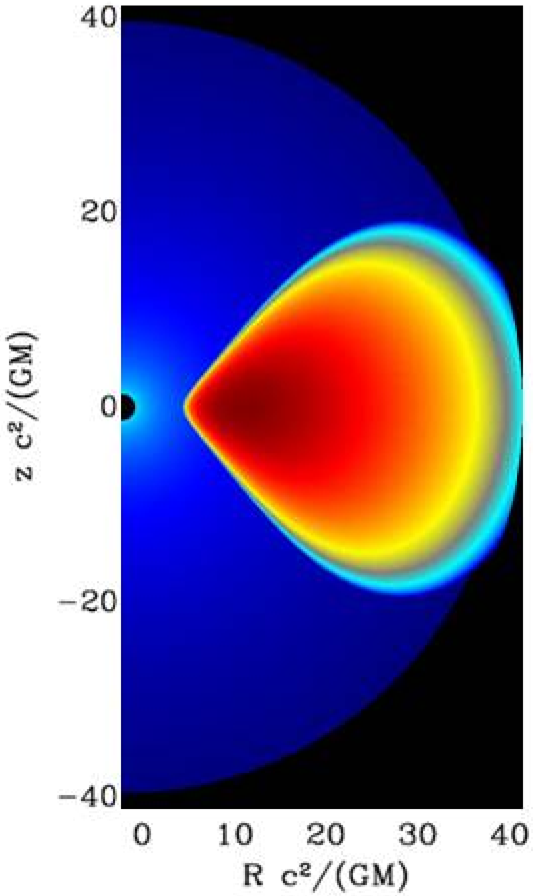
GRMHD Simulations

KerrBH, a = 0.9



0 (non-conservative) 40

(Hawley, DeVilliers 03)



(Hawley, DeVilliers, Gammie, McKinney, Mizuno 2003)

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Existing codes

GRMHD

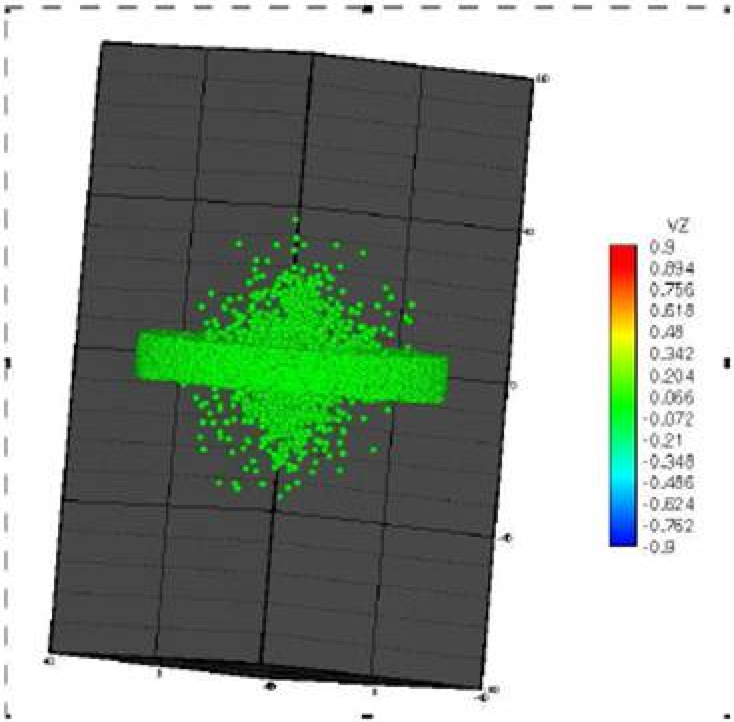
• 1975 – Wilson, J.	acceleration, grav. collapse
• 1993 – Yokosawa	KerrBH acc.
• 1999 – Koide et al.	KerrBH acc. + jet
• 2002 – De Villiers & Hawley	KerrBH acc. + jet
• 2003 – Gammie et al.	KerrBH acc. + jet
• 2004 – Komissarov	BZ process
• 2005 – Shibata & Sekiguchi	KerrBH acc. + jet
• 2005 – Duez et al.	dynamical space-time + GW
• 2005 – Anninos et al.	KerrBH acc.+jet
• 2006 – Anton et al.	KerrBH acc.+jet
• 2006 – McKinney	KerrBH acc.+jet
• 2007 – Giacomazzo & Rezzolla	dynamical space-time + NS evolution

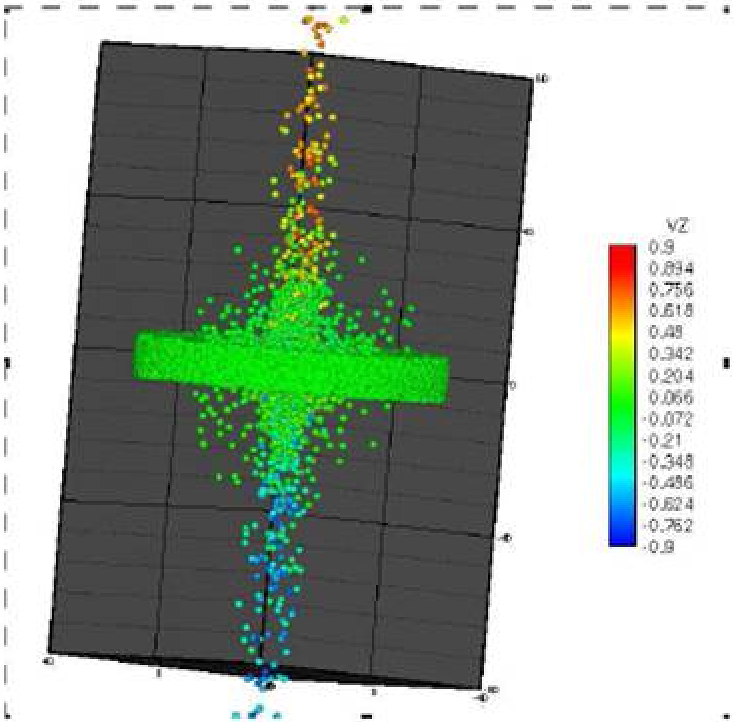
(This list is may not be exhaustive)

GRPIC

• 2007 – Nishikawa, Mizuno, Watson et al. (submitted) KerrBH acc. + jet + something..

GRPIC Simulations



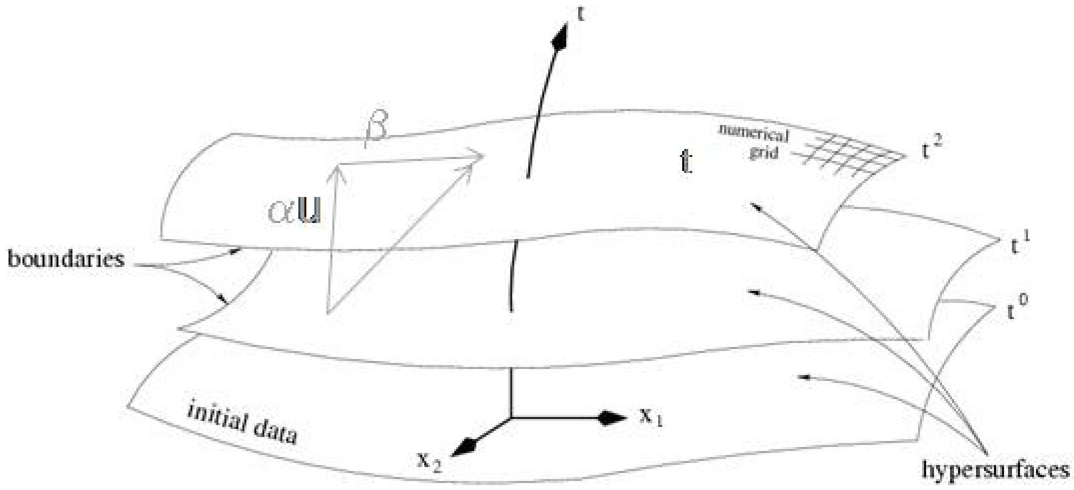


The 3D views of disk and corona particles are shown at $t/\tau_S = 0$ and 1134 ($\tau_S = r_S/c$).

Particle pairs are moving through the jet at different velocities. The jet has a structure wich forms spirals around the z (central) axis. Nishikawa, Mizuno, Watson et al. astro-ph/0612328

GRPIC from scratch

Review



ADM metric:

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

FIDO: $u_\mu = (-\alpha, 0, 0, 0)$

Reconstruct the electromagnetic field, the charge-current vector and the 4-potential:

$$\mathcal{F}^{\mu\nu} = u^\mu E^\nu - u^\nu E^\mu + \varepsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

$$\mathcal{J}^\mu = \rho u^\mu + J^\mu, \quad \mathcal{A}^\mu = \phi u^\mu + A^\mu$$

GRPIC from scratch

System of equations:

Covariant Maxwell equations and induction equation:

$$\mathcal{F}^{\mu\nu}{}_{;\nu} = 4\pi \mathcal{J}^{\mu}, \quad \mathcal{F}_{[\mu\nu;\rho]} = 0, \quad \mathcal{J}^{\mu}{}_{;\mu} = 0$$

Equation of Motion (geodesic equation):

$$m\left(\frac{dv^{\mu}}{d\tau} + \Gamma^{\mu}{}_{\nu\rho} v^{\nu} v^{\rho}\right) = e\mathcal{F}^{\mu\nu} v_{\nu}$$

mass m

charge e

proper time τ

4-velocity v^{μ}

GRPIC from scratch

The 3+1 Maxwell equations in the universal frame:

$$div\mathbf{B} = 0,$$

$$\frac{1}{\sqrt{\gamma}}\partial_t(\sqrt{\gamma}B^i) + [\mathbf{rot}(\alpha\mathbf{E}+\beta\times\mathbf{B})]^i = 0$$

$$div\mathbf{E} = 4\pi\rho$$

$$-\frac{1}{\sqrt{\gamma}}\partial_t(\sqrt{\gamma}E^i) + [\mathbf{rot}(\alpha\mathbf{B}+\beta\times\mathbf{E})]^i = 4\pi\alpha J^i$$

and the induction equation:

$$\frac{1}{\sqrt{\gamma}}\partial_t(\sqrt{\gamma}\rho) + div(\alpha\mathbf{J}) = 0$$

GRPIC from scratch

Equation of Motion (geodesic equation):

$$m\left(\frac{dv^\mu}{d\tau}+\Gamma^\mu{}_{\nu\rho}v^\nu v^\rho\right)=eF^{\mu\nu}v_\nu$$

gamma factor $\mathbf{W}=1/\sqrt{1-\gamma_{ij}V^iV^j}$

$$v^0\equiv\frac{W}{\alpha},$$

"transport" velocity $\mathbf{V}^i\equiv\dfrac{v^i+\beta^i v^0}{W},$

$$V^2\equiv\gamma_{ij}V^iV^j$$

After some nasty calculations we obtain:

$$\frac{d}{d\tau}\left[W\left(-\alpha+\beta\cdot V\right)\right]-\frac{W^2}{\alpha}\left(-\partial_0\alpha+\frac{\alpha}{2}V^iV^j\partial_0\gamma_{ij}+V\cdot\partial_0\beta\right)=\frac{eW}{m\alpha}E\cdot V$$

$$\frac{d}{d\tau}(WV_i)-\frac{W^2}{\alpha}\left(-\partial_i\alpha+\frac{\alpha}{2}V^jV^k\partial_i\gamma_{jk}+V\cdot\partial_i\beta\right)=\frac{eW}{m\alpha}\left[\alpha E+V\times B-(E\cdot V)\beta\right]_i$$

GRPIC from scratch

Weak gravitational field

$$\frac{d}{d\tau}\left[W\left(-\alpha+\boldsymbol{\beta}\cdot\boldsymbol{V}\right)\right]-\frac{W^2}{\alpha}\left(-\partial_0\alpha+\frac{\alpha}{2}V^iV^j\partial_0\gamma_{ij}+\boldsymbol{V}\cdot\partial_0\boldsymbol{\beta}\right)=\frac{eW}{m\alpha}\boldsymbol{E}\cdot\boldsymbol{V}$$

$$\frac{d}{d\tau}\left(WV_i\right)-\frac{W^2}{\alpha}\left(-\partial_i\alpha+\frac{\alpha}{2}V^jV^k\partial_i\gamma_{jk}+\boldsymbol{V}\cdot\partial_i\boldsymbol{\beta}\right)=\frac{eW}{m\alpha}\left[\alpha\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B}-\left(\boldsymbol{E}\cdot\boldsymbol{V}\right)\boldsymbol{\beta}\right]_i$$

weak field limit: $\alpha\to 1,\ \Gamma^{\mu}{}_{\nu\rho}\to 0,\ \beta^i\to 0,\ \gamma_{ij}\to\eta_{ij}$

special relativistic equation of motion:

$$\frac{dW}{d\tau}=-\frac{eW}{m}\boldsymbol{E}\cdot\boldsymbol{V}$$

$$\frac{d}{d\tau}\left(WV_i\right)=\frac{eW}{m}\left(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B}\right)$$

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Theory versus Observation

Theory meets observations I

- funnel-wall jet with evacuated interior, „hollow jet“ (De Villier et al. 2002)
- BZ model fits better obs. Constraint than BP scenario (Blandford & Znajek 1977)
- thin Keplerian disk generates two jets (Mizuno et al. 2006, astro-ph/0609344)

- VLBI of M87 shows asymmetric edge-brightening indicating hollow jet (Krichbaum et al. 2005)
- small size of jet base (Krichbaum et al. 2005)
- Would explain inner, denser jet closer to z-axis

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Theory versus Observation

Theory meets observations II

- helical KH instabilities induced from rotations at jet base
- MHD Kink instabilities (Appl 1996)
- Kink conditions not satisfied (McKinney 2006)
- collimation due to hot corona by p_{rad} (Hawley SAF talk); by Lorentz forces; by disk wind (McKinney 2006)

- S-shaped jet structures, e.g. 3C 120 (Krichbaum et al. 2006)
- explains sudden curved jets
- stability of jets
- collimated jets

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Theory versus Observation

Theory meets observations III

- disk precession caused by Lense-Thirring or binary SMBH system

  [Mizuno et al. 2006:

  initial precessional perturbation propagates through jet as helical structure]

- helical jet structures (e.g. Britzen et al. 2001, Lobanov & Roland 2005)

GRPIC simulations **???**

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Outlook

- Initial & Boundary Conditions (Wald solution, radiative BC)
- implement special relativity into the code, produce some papers :-)
- implement GR in the SRPIC code
- (Ray-Tracing / GR Radiation Transfer equations)
  - task for a poor diploma student.
- and finally, perform a bunch load of simulations…