Astrophysical consequences of a (4+1) dimensional spacetime

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Overview

- Motivation for a (4+1)-dimensional spacetime
- The Randall-Sundrum scenaria
- Consequences for the deflection of light in 4D
- Consequences for the advance of a perihelion/periastron in 4D
- The slowdown of period of the Hulse–Taylor Pulsar PSR1913+16 assuming a 5d Minkowski background

Motivation for a (4+1)-dimensional spacetime

- there are two fundamental energy scales
- electroweak scale

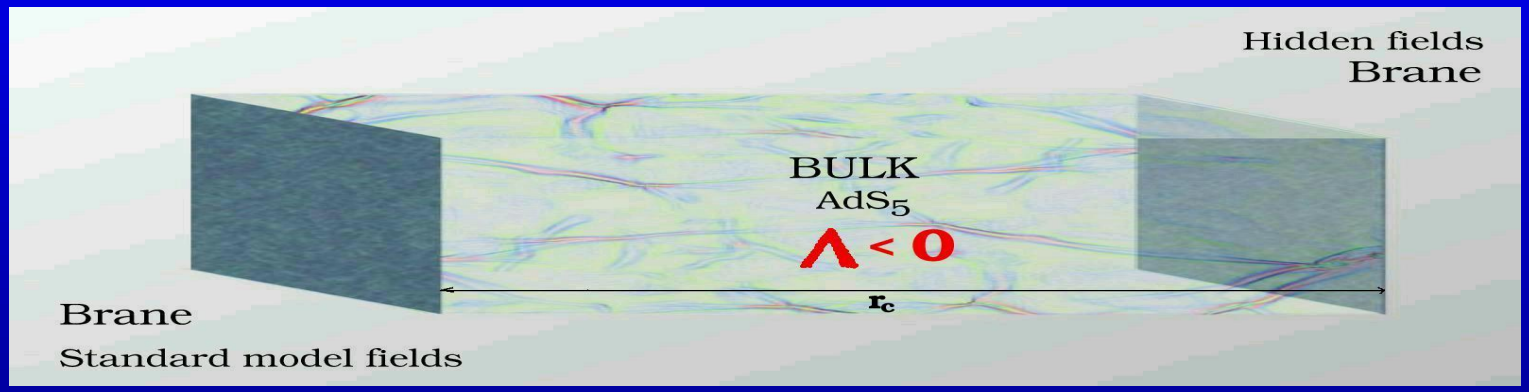
$$m_{EW} \approx 246 \text{ GeV}$$

- Planck scale

$$m_{Pl} \approx 1.2 \times 10^{19} \text{ GeV}$$

- Why is gravity weak compared to the others?

Randall Sundrum models



- two models solve the hierarchy problem with

two branes	one brane
compact	noncompact
graviton in the bulk	graviton ”bound” to the brane

- 4D Planck mass depends on compactification radius r_c

$$M_{PL}^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

L.Randall, R. Sundrum; Phys.Rev. Lett. 1999; **83**, p. 3370 and p. 4690 hep-ph/9905221 and hep-ph/9906064

Cosmological constrains from the Randall Sundrum II model

S.W.Hawking, Das Universum in der Nußschale

Hoffmann und Campe, 2002

- Einstein equation in 5D

$$\tilde{G}_{AB} = \kappa_5^2 \tilde{T}_{AB} - \Lambda_5 \tilde{g}_{AB}$$

- projected Einstein equation in 4D

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu} - \Lambda_4 g_{\mu\nu} + \kappa_5^4 \pi_{\mu\nu} - E_{\mu\nu}$$

- relation between the cosmological constants in 4D and 5D

$$G_N = \frac{\kappa_5^4 \sigma}{48\pi}$$

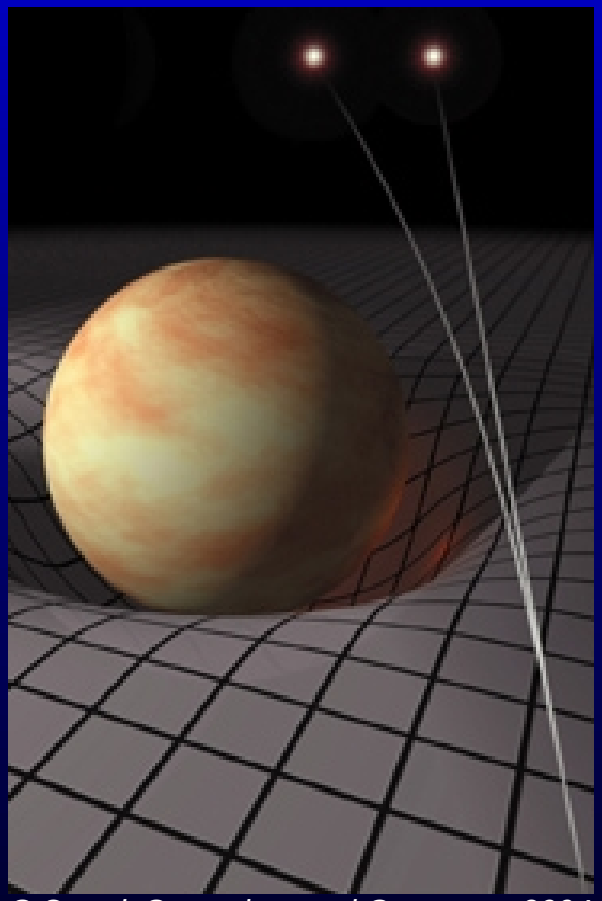
$$\Lambda_4 = \frac{1}{2}\kappa_5^2 \left(\Lambda_5 + \frac{1}{6}\kappa_5^2 \sigma^2 \right)$$

T. Shiromizu, K. Maeda, M. Sasaki, gr-qc/9910076

Deflection of light in 4D

- Schwarzschild-Kottler spacetime

$$ds^2 = (1-\frac{2m}{r}-\frac{1}{3}\,\Lambda\, r^2)\,dt^2-\frac{dr^2}{1-\frac{2m}{r}-\frac{1}{3}\,\Lambda\, r^2}-r^2\,d\Omega_2^2$$



S.Carrol, Spacetime and Geometry, 2004

- „Λ-dependence" from 1^{st} integral of motion for null geodesics

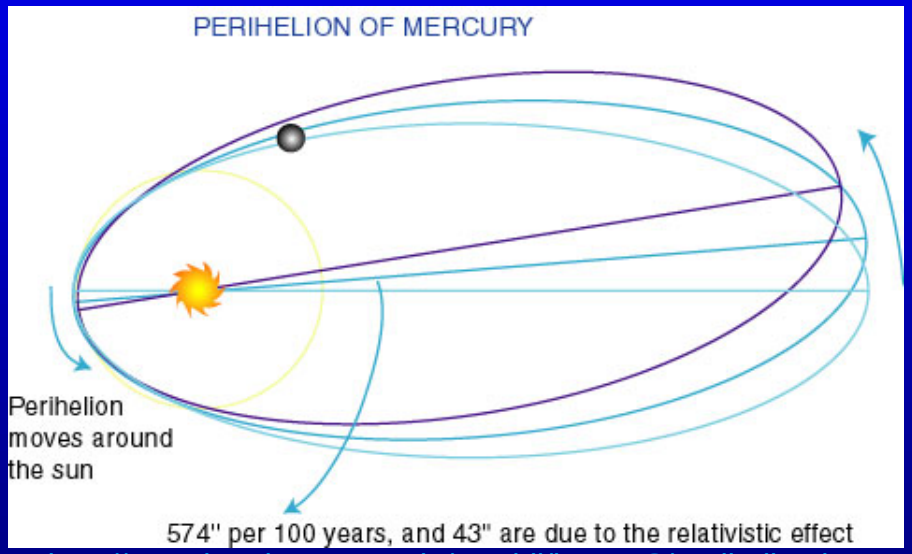
$$\delta = 4\sqrt{\left(\frac{m}{b}\right)^2+\frac{\Lambda m^2}{3}}$$

- TOV eqs. motivate to fix turning point and the source and to consider $\Lambda = 0$ and $\Lambda \neq 0$

$$\delta_{\Lambda=0} = \delta_{\Lambda\neq 0}$$

K. Lake; Physical Review D, vol. 65, Issue 8, id. 087301; gr-qc/0103057

The advance of perihelion/periastron in 4D



http://www.bun.kyoto-u.ac.jp/ suchii/intro.PS/perihelion.jpg

- solve in Kottler spacetime the time-like geodesic

$$\left(\frac{du}{d\phi}\right)^2 = \frac{2GM_\odot}{c^2}\,u^3 - u^2 + \frac{2GM_\odot}{c^2\,L^2} - \frac{1}{3}\left(\frac{1}{u^2L^2}+1\right)\Lambda + \left(\frac{E^2-1}{L^2}\right)$$

- defines a hyperelliptic integral

$$\Delta(\mathrm{obs}) \qquad \Delta(\Lambda = 10^{-56}\ \mathrm{cm}^{-2})$$

$$42.''980 \pm 0.''002 \qquad 42.''9776$$

G.V. Kraniotis, S.B. Whitehouse Class.Quant.Grav.20 (2003) 4817-4835 arXiv:astro-ph/0305181

The Hulse-Taylor Pulsar PSR1913+16

- PSR1913+16 is a binary star system of neutron stars which emmits gravitational waves
- consider a (4+1)-dim. cylinder with a rolled up spatial dimension (RSI-scenario) and the 5D Minkowski is perturbed
- for PSR1913+16 the slowdown of period

$$\dot{\mathrm{T}}_{\mathrm{tens}} = -2.40 \times 10^{-12} \qquad \dot{\mathrm{T}}_{\mathrm{tens+scal}} = -2.88 \times 10^{-12}$$

4D 5D

- but the experimental value is

$$\dot{\mathrm{T}}_{\mathrm{exp}} = -(2.408 \pm 0.01) \times 10^{-12}$$

- the (4+1)-dim. theory predicts a value differing about 20%

R. Durrer, P. Kocian, Classical and Quantum Gravity, Volume 21, Issue 8, pp. 2127-2137 (2004); hep-th/0305181

Summary

- assuming a large extra dimension and a nonzero cosmological constant
  - does not influence the deflection of light
  - is in agreement with the shift of perihelion of Mercury for $\Lambda = 10^{-56}\ \mathrm{cm}^{-2}$
- a compactified extra dimension leeds to a change of the EinsteinQuadrupol Formula